Problem 1 FM versus PM Waveforms (20 points)

Sketch the phase modulation (PM) and frequency modulation (FM) signals that are produced by the sawtooth waveform $m(t)$ drawn below:
Problem 2  Bandwidth of a FM Signal (10 points)

A 10 MHz carrier signal is frequency modulated by a sinusoidal signal of unity amplitude and with a FM factor \( k_f = 10 \text{ Hz/V} \). Find the approximate bandwidth of the frequency modulated signal if the modulating frequency (single tone) is 10 kHz.

Answer:
Start with the expression:
\[
\varphi_{FM}(t) = A_c \cos \left( (2\pi f_c t) + \frac{k_f A_m}{f_m} \cos(2\pi f_m t) \right)
\]

Modulation Index \( \beta = \frac{k_f A_m}{f_m} = \frac{(10 \text{ Hz/V})(1 \text{ V})}{10^4 \text{ Hz}} = 10^{-3} \)

So this is clearly a narrowband FM (NBFM) case. Since it is NBFM we can use the equation that the bandwidth is approximately twice the modulating time frequency \( f_m \).

NBFM bandwidth \( B_T = 2f_m = 20 \text{ kHz} \)

Problem 3  Bandwidth of a FM Signal (10 points)

A 100 MHz carrier signal is frequency modulated by a sinusoidal signal of 75 kHz, such that \( \Delta f = 50 \text{ kHz} \). Find the approximate bandwidth of the frequency modulated signal.

Answer:
We first check to see if this is narrowband FM or wideband FM. To do this we calculate the modulation index \( \beta \) to see if \( \beta \) is much less than one radian, or if it is greater.

Modulation Index \( \beta = \frac{\Delta f}{B} = \frac{50 \text{ kHz}}{75 \text{ kHz}} = 0.667 \)

Therefore, \( \beta \) is not much less than one radian, so it is wideband FM (WBFM), and the bandwidth is given by

WBFM bandwidth \( B_T = 2B(\beta + 1) = 2(7.5 \times 10^4 \text{ Hz})(1.667) = 250 \text{ kHz} \)

Problem 4  (10 points)

You are given the equation below for a FM waveform,
\[
\varphi_{FM}(t) = 100 \cos \left[ (2\pi \times 10^5 t) + 35 \cos(100\pi t) \right]
\]

From this expression, estimate the bandwidth of the FM signal.
Answer:
Starting with the formula, \( \varphi_{FM}(t) = A_c \cos[(2\pi f_C t) + \beta \cos(2\pi f_m t)] \)
we immediately see can equate terms where \( \beta = 35 \) and \( f_m = 50 \text{ Hz} \) (from \( 2\pi f_m = 100\pi \)),
Now we find the frequency deviation \( \Delta f = \beta f_m = 35(50 \text{ Hz}) = 1750 \text{ Hz} \).

\[
WBFM \text{ bandwidth } B_r = 2(\Delta f + B) = 2(\Delta f + f_m) = 2(1750 + 50) = 3600 \text{ Hz}
\]

**Problem 5 Multiplexed Signals** (20 points)

You have two signal sources, one of frequency of \( (f_c + \Delta f) \) and the other of \( (f_c - \Delta f) \), which can be combined into a output \( y(t) \) as controlled by a switch. The switch is being toggled at a modulating rate of frequency \( f_m \). Sketch the approximate waveform \( y(t) \) at the output of the circuit. Note: This is time multiplexing the signals.

\[
A \cdot \cos(2\pi (f_c + \Delta f)t)
\]

\[
A \cdot \cos(2\pi (f_c - \Delta f)t)
\]

Answer:
The switch alternates between the two frequencies so that the waveform will look something like the sketch below.
Problem 6 Frequency Modulation (20 points)

A frequency modulator is supplied with a carrier at frequency $f_c = 5$ MHz and an audio tone signal of 1 volt amplitude and frequency $f_m = 1$ kHz. It produces a frequency deviation $\Delta f$ of 10 kHz as the output of the frequency modulator.

(a) The FM waveform described above is passed through a series of frequency multipliers with a total multiplication factor of 12, that is, $f_{out} = 12f_c$. After passing through the frequency multipliers what is the frequency deviation $\Delta f$ at the output?

Answer:
The multiplier applies to both $f_c$ and to $\Delta f$, thus, the increase in the frequency deviation is

$$\Delta f \text{ after multiplication is } \Delta f = 12 \cdot \Delta f = 12 \cdot 10 \text{ kHz} = 120 \text{ kHz}$$

(b) The output of the multiplier in part (a) is next input to a mixer stage with the LO oscillator frequency set at $f_{LO} = 55$ MHz. Find the sum-frequency output $f_o$ of the mixer's IF port and also find the magnitude of the frequency deviation $\Delta f$.

Answer:
A mixer produces both sum and difference frequencies at the IF port. The multiplied carrier frequency $f_c$ input to the RF port is 60 MHz, hence, the sum frequency is 60 MHz plus 55 MHz = 115 MHz and the difference frequency is 60 MHz minus 55 MHz = 5 MHz. A mixer simply frequency translates a signal so the frequency deviation $\Delta f$ remains 120 kHz as found in part (a) above. Therefore, $\Delta f = 120$ kHz.

(c) What is the modulation index $\beta$ at the output of the modulator.

Answer:
The modulation index $\beta$ is simply

$$\beta = \frac{\Delta f}{f_m} = \frac{120 \text{ kHz}}{12 \text{ kHz}} = 10$$

(d) Next, we change the audio input to the modulator to $V_m = 2$ volts and the modulation tone frequency $f_m$ is reduced to 500 Hz at the modulator’s output. What is the modulation index $\beta$ and the frequency deviation $\Delta f$?
Answer:
The amplitude of the modulating voltage \( V_m \) is increased from 1 volt to 2 volts and the modulating tone frequency \( f_m \) is reduced to 500 Hz (not the typographical error in the problem set which stated 500 kHz).

We are asked to find the modulation index \( \beta \) and the frequency deviation \( \Delta f \)

\[
\Delta f = k_f A_m, \quad \text{and} \quad \beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m}
\]

Now we see that \( \Delta f \) is doubled from its prior value, namely, \( \Delta f = 2k_f \) and

\[
\beta = \frac{\Delta f}{f_m} = \frac{k_f A_m}{f_m} = \frac{2k_f}{500 \text{ Hz}},
\]

but 500 Hz is one-half of the prior \( f_m \), so \( \beta = 4k_f \).