Signals
Some Preliminary Discussion

ES 442 Analog & Digital Communication Systems
Lecture 2
Complex Exponentials to Represent Sine & Cosine

\[
\cos(2\pi ft) = \frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2}
\]

\[
\sin(2\pi ft) = \frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j}
\]
Forming a Cosine Wave Signal

\[ \cos(2\pi ft) = \frac{1}{2} e^{j2\pi ft} + \frac{1}{2} e^{-j2\pi ft} \]
Forming a Sine Wave Signal

Counter rotating vectors (phasors)

\[-e^{-j2\pi ft}, e^{j2\pi ft}\]

\[\sin(2\pi ft) = \frac{1}{2j} e^{j2\pi ft} - \frac{1}{2j} e^{-j2\pi ft}\]

Projection onto imaginary-axis:

Amplitude

Time \(t\) evolution
How Do We Explain Negative Frequencies?

“The existence of the spectrum at negative frequencies is somewhat disturbing to some people because, by definition, the frequency (number of repetitions per second) is a positive quantity. How do we interpret a negative frequency \(-f_0\)?

Lathi & Ding
Pages 64-66
Analog Signals versus Digital Signals

Our viewpoint:

An **Analog Signal** is amplitude versus time where its amplitude can take on an infinite number of values. A **Continuous-Time Signal** is a signal that is continuous-valued over time. Analog signals encode data, a message, or information, using variation in amplitude, generally restricted to an interval of real numbers. An analog signal is continuous-time, but a continuous-time signal does not have to be analog. Therefore, a continuous-time signal may be either an analog signal or a digital signal (*i.e.*, a stream of discrete multiple values).

A **Digital Signal** is a discrete-time signal taking on discrete values within a sequence of discrete intervals (*i.e.*, symbol periods). A digital signal can range from a binary bitstream carried by a continuous-time signal to a sequence of digital words representing data, a message, or information, contained within the signal.
## Distinguishing Signals from Waveforms

<table>
<thead>
<tr>
<th>Analog Signals</th>
<th>Digital Signals</th>
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<tbody>
<tr>
<td>(1) A parameter of the signal represents a physical parameter</td>
<td>(1) Represents a sequence of numbers or states</td>
</tr>
<tr>
<td>(2) Parameter is time-varying</td>
<td>(2) Numbers can change in discrete time (said to be time-varying)</td>
</tr>
<tr>
<td>(3) Parameter takes on any value within a defined range (said to be continuous)</td>
<td>(3) Numbers restricted a finite set of discrete values</td>
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Waveforms:

- **Analog signal**
- **Digital signal**

All signal waveforms are analog – the difference is what they represent!
Analog Signals vs. Digital Signals: Examples

**Analog & continuous**

**Analog & discrete**

**Digital & continuous**

**Digital & discrete**
QUESTION: How would you classify these waveforms?

(1) Continuous, (2) Discrete, (3) Analog, (4) Digital
Importance of Time Variation in Communication Signals

Example: Bit Sequence of 10101010101 . . .

High state Represents a “1”
Low state Represents a “0”

Not communication. Why?

This is a **periodic** waveform.

A pure sinusoidal waveform can not transmit data!

Example: Bit Sequence of 10001010111 . . .

“Data” can be transmitted. Why?

Not a **periodic** waveform.
Classifications of Signals

**Deterministic**
- Periodic: Sinusoidal, Triangular, Rectangular
- Aperiodic: Transient, Unit pulse response
- Quasi-periodic: ECG waveform, Temperature record

**Random (Stochastic)**
- Stationary: Noise in Electronic Circuits
- Non-stationary: Language, Music, etc.

**Mathematical representation possible**
- Often can calculate the waveform
- Roughly approximate mathematically

**Not mathematically calculable**
- White Gaussian Noise
- Voice waveform
**Bandwidth**

The bandwidth of a signal provides a measure of the extent of significant spectral content of the signal for positive frequencies. What does significant mean?

1. **3-dB Bandwidth** – The separation (along positive frequency axis) between the points where the amplitude drops to $\frac{1}{\sqrt{2}}$ of its peak value.

2. **Null-to-null Bandwidth** – For example, for the sinc function the bandwidth would be the frequency width from $-1/T$ to $1/T$ (null-to-null points).

3. **Root-mean-square (RMS) Bandwidth** – Defined to be

$$BW_{RMS} = \sqrt{\frac{\int_{-\infty}^{\infty} f^2 |G(f)|^2}{\int_{-\infty}^{\infty} |G(f)|^2}}$$

And still there are other bandwidth definitions!
Signal Energy and Power

For a real signal $g(t)$ the signal energy $E_g$ is defined to be

$$E_g = \int_{-\infty}^{\infty} g^2(t) dt$$

For a complex signal $g(t)$ the signal energy $E_g$ is defined to be

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt$$

The signal power is more useful. For a real or complex signal $g(t)$ the signal power $P_g$ is defined to be

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt$$

The signal power $P_g$ is the time average (mean) of the signal amplitude Squared (sometimes called the “mean-square” value of $g(t)$).
Parseval’s Theorem

The Fourier transform of $g(t)$ is $G(f)$. The signal energy is related to the $G(f)$ by Parseval’s theorem:

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 \, df$$

The quantity $|G(f)|^2$ is called the energy spectral density of $g(t)$ at frequency $f$ and its graph is the energy spectrum of $g(t)$.

Energy spectral density: $|G(f)|^2$ (energy/Hz)
Units of Signal Power

The standard unit for signal power is in watts (W).

Often signal power is expressed on a decibel scale. Strictly speaking, this requires we use ratios of power as indicated by

\[
\text{Decibels (dB)} = 10 \log_{10}(P_2/P_1)
\]

It is common practice to express power relative to a reference power. For example, sometimes say we use 1 watt as the reference power. Then, we can use the above equation with \( P_1 = 1 \text{ watt} \), so that power level \( P_2 \) in watts is expressed on a decibel scale by

\[
P_2 \text{ (in dBW)} = 10 \log_{10}(P_2/1) = 10 \log_{10}(P_2) \text{ dBW}
\]

where \( P_2 \) is in watts (the unit of watts is cancelled by the denominator of 1 watt) and taking \( 10 \log_{10} \) of the power ratio gives \( P_2 \) in dBW rather than in watts. If instead \( P_1 \) is one milliwatt (1 mW), then \( P_2 \) is expressed in units of milliwatts and the decibel scale is in units of dBm. Hence,

\[
P_2 \text{ (in dBm)} = 10 \log_{10}(P_2/1 \text{ mW}) = 10 \log_{10}(P_2) \text{ dBm}
\]

Why is it convenient to use a decibel scale in expressing power levels?
Questions