Amplitude Modulation – Early Radio
ES 442 – Spring 2016
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\[ f_{LO} \gg f_{\text{audio baseband}} = f_m \]
Modulation Options

Continuous Wave Modulation

Linear Modulation

Non-Linear Modulation

Pulse Wave Modulation

Amplitude Modulation

Frequency Modulation

Phase Modulation

AM Modulation -- Radio
Baseband versus Carrier Communication

**Baseband communication** is the transmission of a message as generated.

**Carrier communication** requires the modulation of a message onto a carrier signal to transmit it over a different frequency band. We will use modulators to do the frequency translation.

(Note: “Pulse modulated” signals, such as PAM, PWM, PPM, PCM and DM are actually baseband digital signal coding (and not the result of frequency conversion).

**Use of Sinusoidal Carriers**: With a sine waveform there are 3 parameters which we can use to “modulate” a message onto the carrier – they are the **amplitude, frequency** and **phase** of the sinusoid.
Amplitude Modulation Definition

*Amplitude modulation* (AM) is a modulation technique in which the *amplitude* of a high frequency sine wave (typically at a *radio frequency*) is varied in direct proportion to a *modulating signal* $m(t)$. The modulating signal contains the intended message or information – sometimes consisting of audio data, as in radio broadcasting or two-way radio communications.

The high frequency sinusoidal waveform (*i.e.*, *carrier*) is modulated by combining it with the modulating signal using a *multiplier* or *mixer*. 
Amplitude Modulation in Pictures

**Frequency Domain**

- Tone modulated AM signal
- Voice modulated AM signal
- 100 kHz carrier modulated by a 5 kHz audio tone
- 100 kHz carrier modulated by an audio signal (frequencies up to 6 kHz)

**Time Domain**

- Carrier Signal
- 5 kHz Audio tone
- Modulating Sine Wave Signal
- Amplitude Modulated Signal
Example: Voice Signal – 300 Hz to 3400 Hz Baseband

$m(t)$ represents the source message signal.

Time Domain
Voice Band for Telephone Communication

Voice Channel
0 Hz – 4 kHz

Voice Bandwidth
300 Hz – 3.4 kHz

Frequency Domain
Representative Voice Spectrum for Human Speech

For the telephone AT&T determined many years ago that speech could be easily recognized when the lowest frequencies and frequencies above 3.4 kilohertz were cutoff.

Waveform as received from a microphone converting acoustic energy into electrical energy.

Fast Fourier transform of the above speech waveform showing energy over frequency from 0 Hz to 10 kHz.
Early AM Crystal Radio Receiver (Bare Minimum)

A crystal radio receiver, also called a crystal set or cat's whisker receiver, is a very simple radio receiver, popular in the early days of radio. It needs no other power source but that received solely from the power of radio waves received by a wire antenna. It gets its name from its most important component, known as a crystal detector, originally made from a piece of crystalline mineral such as galena. This component is now called a diode.

Note: 1N34A is a germanium diode

Earphones

LC Tuned Circuit
Foxhole Radio (from World war I)

- Cold water pipe
- Coil – 120 turns of wire
- Ground
- Razor blade
- Safety pin
- Aerial connection
- Pencil point
- Earphones

http://bizarrelabs.com/foxhole.htm
Crystal Radio Receiver from 1922

Diagram from 1922 showing the circuit of a crystal radio. This common circuit did not use a tuning capacitor, but used the capacitance of the antenna to form the tuned circuit with the coil.

Galena (lead sulfide) was probably the most common crystal used in cat's whisker detectors.
Double-Sideband Amplitude Modulation

Let $m(t)$ represent the source message signal. The Fourier transform pair is $m(t) \Leftrightarrow M(f)$. We make use of the frequency shifting property, Let $s(t) = m(t) \cos(2\pi f_c t)$, therefore the FT pair is

$$m(t) \cos(2\pi f_c t) \Leftrightarrow \frac{1}{2} \left[ M(f + f_c) + M(f - f_c) \right]$$

Lathi & Ding
Page 180
Double-Sideband Amplitude Modulation

Frequency-shifting property

\[ m(t) \]

\[ m(t) \cos(2\pi f_c t) \]

\[ \cos(2\pi f_c t) \]

\[ m(t) \cos(2\pi f_c t) \]

\[ M(f) \]

\[ USB \]

\[ LSB \]

\[ f_c \]

\[ 2B \]

Lathi & Ding
Page 181
Double-Sideband Suppressed Carrier Demodulation

Output of mixer is \( e(t) = m(t) \left[ \cos(2\pi f_c t) \right]^2 \)
\[
e(t) = \frac{1}{2} m(t) \left[ 1 + \cos(2 \cdot (2\pi f_c) t) \right], \text{ taking FT gives}
F[e(t)] = \frac{1}{2} M(f) + \frac{1}{4} [M(f + 2 f_c) + M(f - 2 f_c)]
\]

No carrier is present

Lathi & Ding pp. 181-182
AM Modulation Index Basics

The amplitude modulation, AM, modulation index can be defined as the measure of extent of amplitude variation on an un-modulated carrier. The modulation index for amplitude modulation indicates the amount by which the modulated carrier varies around its un-modulated level.

When expressed as a percentage it is the same as the depth of modulation. In other words it can be expressed as:

\[
\text{Modulation Index} \quad \mu = \frac{M}{A}
\]

where \( A \) is the carrier amplitude, and
\( M \) is the modulation amplitude (peak change in the RF amplitude relative to its un-modulated value.

Example: AM modulation index of 0.5, the modulation causes the signal to increase by a factor of 0.5 and decrease to 0.5 of its original level.

\[
y(t) = [1 + m(t)] \cdot A \cos(2\pi f_c t)
\]
AM Modulation Index Basics - II

50% Modulation

100% Modulation

150% Modulation
DSB-SC Modulation & Demodulation of Cosine

Single tone modulation is \( \cos(2\pi f_m t) \)

Modulating signal spectrum

\[
\frac{1}{2} \left[ \cos \left( (2\pi f_c + 2\pi f_m) t \right) + \cos \left( (2\pi f_c - 2\pi f_m) t \right) \right]
\]

DSB Spectrum

USB

\(- (f_c + f_m)\)

LSB

\(- (f_c - f_m)\)

LSB

\(f_c - f_m\)

USB

\(f_c + f_m\)

Suppress by low-pass filtering

Demodulated signal spectrum

Lathi & Ding
Page 183
DSB-SC Time Domain Waveforms (Cosine Tone)

\[ g(t) = m(t) \cos(2\pi f_c t) \]

\[ e(t) = \frac{1}{2} m(t) \left[ 1 + \cos(2(2\pi f_c) t) \right] \]

\[ e(t) \text{ low-pass filtered (} \frac{1}{2} m(t) \text{)} \]
Categories of Amplitude Modulation

- Baseband spectrum
- Conventional AM (aka Full AM)
- Double-Sideband-Suppressed Carrier (DSB-SC)
- Single-Sideband /Upper Sideband SSB/USB
- Single-Sideband /Lower Sideband SSB/LSB
- Also Vestigial Sideband and Amplitude Companded SSB
Modulators – Types of Modulators

1. **Electronic multipliers** used for modulation

   Integrated circuits have made it easy to buy inexpensive multipliers operating to very high frequencies with power gain.

2. **Nonlinear component modulators**

   Almost any nonlinearity will work, but a very inexpensive but strongly nonlinear device is the diode. Transistors are also nonlinear and work well as modulators.

3. **Switching modulators**

   Switching is an easily attained function with diodes and transistors in electronic circuits – especially in integrated circuits.
Multipliers Used for Modulation

Multiplier Example: \( g(t) \) and \( s(t) \) are sinusoidal inputs.

If \( g(t) = A_{RF} \sin(\omega_{RF} t) \) and \( s(t) = B_{LO} \sin(\omega_{LO} t) \), then

\[
y(t) = g(t) \times s(t) = A_{RF} B_{LO} \sin(\omega_{RF} t) \sin(\omega_{LO} t) \]

\[
= \frac{1}{2} A_{RF} B_{LO} [\sin(\omega_{RF} + \omega_{LO})t + \sin(\omega_{RF} - \omega_{LO})t]
\]

- sum frequency
- difference frequency

Sometimes called a “mixer”
Using Nonlinearity For Modulation

\[ V_o = V_{dc} + G V_i + A V_i^2 + B V_i^3 + \ldots \]

Let \( V_i = A_{RF} \sin \omega_{RF} t + B_{LO} \sin \omega_{LO} t \), then we get frequencies,

\[ V_i \Rightarrow \omega_{RF} \text{ and } \omega_{LO} \] (linear)

\[ V_i^2 \Rightarrow (\omega_{RF} + \omega_{LO}), (\omega_{RF} - \omega_{LO}), 2\omega_{RF} \text{ & } 2\omega_{LO} \] (square law)

\[ V_i^3 \Rightarrow (2\omega_{RF} + \omega_{LO}), (2\omega_{RF} - \omega_{LO}), (2\omega_{LO} + \omega_{RF}), (2\omega_{LO} - \omega_{RF}), 3\omega_{RF} \text{ & } 3\omega_{LO} \]

\ldots through third order \ldots

Conclusion: Nonlinearity generates new frequencies.
Using Nonlinearity For Modulation and Demodulation

A “hopelessly unsophisticated” mixer.

— Tom Lee (Stanford University)

The unbalanced single-diode mixer has no isolation and no conversion gain.

Single-diode mixers have been used in many applications --

(1) Detectors for radar in WW II
(2) Early UHF Television tuners
(3) Crystal radio detectors
(4) mm-wave & sub-mm-wave receivers
Diode Demodulator

The user has a choice of changing the filter to meet their needs.
Modulation and Demodulation Example

(a) message signal

(b) message after modulation

(c) demodulated signal

(d) recovered message is a LPF applied to (c)
Two conditions must be met for an envelope detector to work:

1. Narrowband [meaning $f_c >> \text{bandwidth of } m(t)$]
2. $A + m(t) \geq 0$
Choosing the RC Time Constant in Envelope Detector

Time constant $RC$ is too short.

Design criteria is $2\pi B < \frac{1}{RC} \ll 2\pi f_c$

Time constant $RC$ is too long.
Rectifier Detection

\[ V_{\text{rect}}(t) = [(A + m(t)) \cos(\omega_c t)] \cdot w(t) \]

\[ = [(A + m(t)) \cos(\omega_c t)] \left\{ \frac{1}{2} + \frac{2}{\pi} \left[ (\cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \ldots \right] \right\} \]

\[ = \frac{1}{\pi} (A + m(t)) \quad + \quad \text{other terms}. \]

Note: Multiplication with \( w(t) \) allows rectifier detection to act essentially synchronous detection without a carrier.

AM Modulation -- Radio 28
Switching Modulator – Generating $m(t)\cos(2\pi f_c t)$

Figure 4.4
Lathi & Ding
Page 186
Generating $m(t)\cos(2\pi f_c t)$ using Convolution

From Convolution Theorem:

$$\mathcal{F}[\mathcal{L}(t) g(t)] \leftrightarrow C_n(f) * G(f)$$

Output Spectrum

$m(t)\cos(2\pi f_c t)$
Example: Diode Ring Modulator

\[ m(t) \]

\[ A \cos(2\pi f_c t) \]

\[ \frac{4}{\pi} \left[ \cos(2\pi f_c t) - \frac{1}{3} \cos(3 \cdot 2\pi f_c t) + \frac{1}{5} \cos(5 \cdot 2\pi f_c t) - \ldots \right] \]

\[ v_i = m(t)w_0(t) \]

Km(t)cos(2πf_ct)

Figure 4.6
Lathi & Ding
Page 188
Double-Balanced Diode Ring Mixer
Commercial Diode Ring Mixer (Mini-Circuits)

Mixer in surface-mount package

Ring of FET devices operated as nonlinear resistances

1. These diagrams show a typical diode ring quad (top) or FET passive mixer (bottom); the inset photograph shows a surface-mount-packaged mixer from Mini-Circuits (Brooklyn, NY).
Gilbert Cell Double-Balanced Mixer

Widely used in microwave bands.
Frequency Conversion – from $f_c$ to $f_I$

Multiplying a modulated signal by a sinusoidal moves the frequency band to sum and difference frequencies.

**Example 4.2:** We want to convert from frequency $\omega_c$ to frequency $\omega_I$.

![Diagram showing frequency conversion](image)

$$m(t) \cdot \cos(\omega_c t) \rightarrow g(t) \rightarrow m(t) \cdot \cos(\omega_I t)$$

Input frequency $f_c$

Output frequency $f_I$

$$2 \cdot \cos((\omega_c \pm \omega_I) t)$$

Note: Super-heterodyning: $\omega_c + \omega_I$; Sub-heterodyning: $\omega_c - \omega_I$
Demodulation of DSB-SC Signals

Demodulation of a DSB-SC signal involves multiplication with the carrier signal – it is identical to modulation. Reference: Figure 4.1 in Lathi & Ding.

At the receiver, the incoming signal is multiplied by a local carrier of frequency and phase in synchronism with the incoming signal. The only difference between the modulator and demodulator lies in input signal and the output filter.

With an incoming signal with suppressed carrier, the receiver must generate a local carrier which is synchronous in phase with the incoming signal. These demodulators are called synchronous or coherent demodulators.

Example 4.3 in Lathi & Ding, page 190.
In conventional AM a carrier is transmitted. Let $m(t) \Leftrightarrow M(f)$. The AM signal is written as

$$\varphi_{AM}(t) = A \cos(2\pi f_c t) + m(t) \cos(2\pi f_c t) = [A + m(t)] \cos(2\pi f_c t)$$

Taking the Fourier transform gives

$$\varphi_{AM}(t) \Leftrightarrow \frac{1}{2} [M(f + f_c) + M(f - f_c)] + \frac{A}{2} [\delta(f + f_c) + \delta(f - f_c)]$$

where we require $[A + m(t)] \geq 0$
Superheterodyne Receiver

AM radio receiver:
Single Sideband (SSB)

DSB-SC is spectrally inefficient because it uses twice the bandwidth of the message.

The signal can be reconstructed from either the upper sideband (USB) or the lower sideband (LSB).

SSB transmits a bandpass filtered version of the modulated signal.
Single Sideband (SSB)

Multiplication of a USB signal by \(\cos(\omega_c t)\) shifts the spectrum to left and right.
Phase-Shift Method to Generate SSB

\[ \varphi_{SSB}(t) = m(t) \cos(\omega_c t) \mp m_h(t) \sin(\omega_c t) \]

where minus sign applies to USB and plus sign applies to the LSB.

\( m_h(t) \) is \( m(t) \) phase delayed by \( \pi/2 \)
Shortwave radio uses upper MF and all of HF (1.8–30 MHz). Transmitter and receiver must agree on use of LSB vs USB. International broadcast bands:

- 120m (2300-2495 kHz): LSB
- 90m (3200-3400 kHz): LSB
- 75m (3900-4000 kHz): USB
- 60m (4750-5060 kHz): LSB
- 49m (5900-6200 kHz): USB
- 41m (7200-7450 kHz): USB

NIST’s WWV broadcasts time from Fort Collins, CO, on frequencies 2.5, 5, 10, 15, and 20 MHz. Each frequency uses a different antenna tower whose length is half the wave length (from 15m to 120m). Effective radiated power (ERP) is 2.4 kW for 2.5 and 20 MHz and 10 kW other frequencies.
Quadrature-Carrier Multiplexing

Quadrature-carrier multiplexing allows for transmitting two message signals on the same carrier frequency.

(1) Two quadrature carriers are multiplexed together,

(2) Signal $m_1(t)$ modulates the carrier $\cos(2\pi f_c t)$, and signal $m_2(t)$ modulates the carrier $\sin(2\pi f_c t)$.

(3) These two modulated signals are added together & transmitted over the channel

$$\varphi_{QAM}(t) = m_1(t)\cos(2\pi f_c t) + m_2(t)\sin(2\pi f_c t)$$
Quadrature Amplitude Modulation and Demodulation

QAM

\[ 2 \cos(2\pi f_c t) \]

\[ 2 \sin(2\pi f_c t) \]
**Quadrature-Amplitude Demodulation**

\[ x_1(t) = \phi_{QAM}(t) \cdot 2 \cos(\omega_c t) \]

\[ = \left[ (m_1(t)) \cos(\omega_c t) + (m_2(t)) \sin(\omega_c t) \right] \cdot 2 \cos(\omega_c t) \]

\[ = 2m_1(t) \cos^2(\omega_c t) + 2m_2(t) \sin(\omega_c t) \cos(\omega_c t) \]

\[ = m_1(t) + m_1(t) \cos(2\omega_c t) + m_2(t) \sin(2\omega_c t) \]

We can recover \( m_1(t) \) by passing \( x_1(t) \) through a LPF.
Power of AM Signals

\[ \varphi_{AM}(t) = (A + m(t)) \cos(\omega_C t) = A \cos(\omega_C t) + m(t) \cos(\omega_C t) \]

Carrier power is

\[ P_C = A^2 \frac{1}{2\pi \omega_C} \int_0^{1/2\pi \omega_C} \cos^2(\omega_C t) \, dt = \frac{1}{2} A^2 \]

Signal power \( P_S \) is \( \frac{1}{2} P_m \), where

Message power uses an "appropriate" interval,

\[ P_m = m^2(t) = \frac{1}{T} \int_{t_0}^{t_0+T} m^2(t) \, dt \]
Pulse Amplitude Modulation (PAM) – Digital Signal

How is PAM in digital communication similar to AM in analog communication?
Elenco AM/FM Dual-Radio Receiver Kit
Questions