COMMON SCIENCE AND MATHS AREAS
MUTARE GIRLS HIGH SCHOOL
MUTARE, ZIMBABWE

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Foreword

Genesis

In 1999, the faculty of Mutare Girls' High School recognized and proposed that the faculty needed an opportunity for on-going professional development at a time and place set apart for such purposes only. At that time a lack of resources prevented further progress, but the matter resurfaced the following year. Specifically, the genesis of this project was the recognition by the staff and Headmaster at an all-faculty staff meeting during the third term, 2000, for the need to address cross-discipline communication and faculty development across the sciences and mathematics.

One of the senior science teachers, Ms. Nyanhanda, undertook to poll teachers about problematic areas among these disciplines, as they perceived them. Her compilation has formed the basis of the table of contents of the workshop materials. PCV, Clement Falbo, began to develop suitable core problems, provisionally titled "Applications in Maths and Science: Problem Set" that would meet the expressed need of the faculty to improve instruction in these areas while PVC Jean Falbo began exploration of relevant logistical, organizational and cost considerations. Ms. Falbo, a volunteer member of the science faculty, will also participate in the specific proposed activities. Mr. Falbo, a mathematician, is currently teaching Computer Application classes.

Preface

This booklet is envisioned as being maximally flexible by being in a loose-leaf notebook and archived on electronic disks. This format permits revisions, expansions and selective use of materials, especially since MGHS has a computer laboratory and printing capabilities.

The theme behind the text is to fill in some of the background thinking that is related to problem solving in maths and the sciences. But, at the same time, each discipline keeps its own identity.

Here is the underlying philosophy. The biological sciences, mathematics and the physical sciences are three atoms tied together in a triple bond by pedagogy, analysis and computation. Pedagogically, in each of these disciplines' teachers usually assign daily work, they apply general principles to specific cases and they develop general theory through experiments and specific examples. Analysis includes logical interpretation of experiments, the development of mathematical and scientific models that adhere to basic laws. Computation includes the use of calculators and computers across the three disciplines to obtain accurate results.

Traditionally, the mathematics teachers and the science teachers each go their separate ways, failing to exploit the opportunities to provide students with a greater understanding of some of the common areas in these three disciplines. If teachers emphasize the connections, the student gains insight and a deeper understanding that should be manifest in her exam marks on the national exams in the respective fields. It is hoped that this booklet will help teachers and students make the connections mentioned above.

For their contributions to this project, thanks are due to the Science and Maths Faculty and to Mr. Kwari, the headmaster at MGHS. We acknowledge the U.S. Peace Corps Office in Harare for supporting this project. A special thanks is due to former MGHS student, Miss Shorai Mukaka, for working out the problems and for contributing some of the problems.

Applications in Maths and Science

CHAPTER 1. TALLIES

Counting is an important activity in science and mathematics as well as in our everyday life. We count the money in our pocket or how many apples we are getting in a bag. Pupils count the number of lessons they are taking or the number of exams they have to write, etc. Teachers count how many papers they have marked and how many remain to be marked.

One of the most convenient way to keep track of what ever it is you are counting is by the use of "talley" marks. such as: | for 1, || for 2, ||| for 3, |||| for 4 and 卌 for 5, etc. What makes tallies convenient is that you can increase the number without having to erase the previous number.

Suppose, for example, in a biology class you are counting the number of each of two kinds of plants (Furry Grass and Black Jack) growing in a patch of ground. It would not be so convenient for you to use ordinary notation 1;2;3; etc. because, say you have already counted 7 Furry Grasses and you come across another one, then you would have to erase the 7 and then write 8. Later if you found another one you would have to erase the 8 and write 9, etc. Your notebook would get pretty messy after a while. But if you had used a talley system you would have had 卌 ||, then when you found the eighth one you could just add another talley mark and get 卌 |||, then later 卌 ||||, etc.

Of course Tallies also have a disadvantage: To write large numbers, in tallies would be awkward, for example try writing 233 in tallies. You would have to fill up a page.

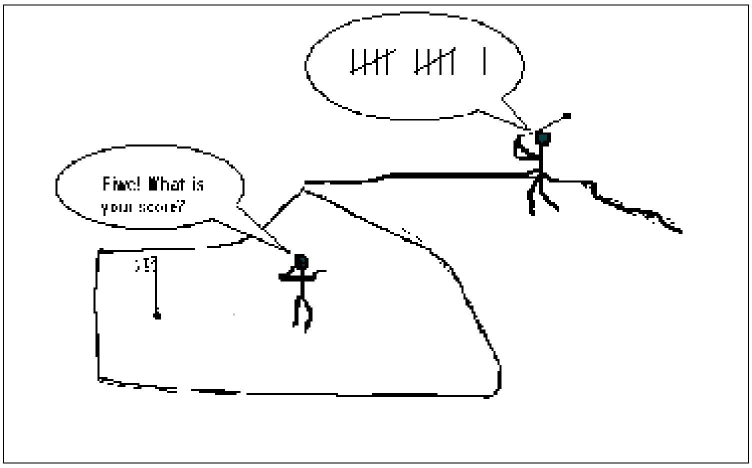

Figure 1.1 Golfing Cartoon

Exercises

1.1 In the cartoon above, write the golfer's score in ordinary notation for numbers.

1.2. A golfer, *Matthew*, kept a talley of his golf strokes (his score) on five holes of golf. Here is his score card talley.

Hole Number	Strokes
Hole 1	||/||
Hole 2	||||
Hole 3	||/|| |
Hole 4	||/|| ||
Hole 5	|||
Total	

Table 1.1

(a) Write his total on all five holes in talley notation.
(b) Convert each hole **and** the total to *ordinary* numerals.

Problem 3 refers to data in problem 2.
1.3. (a) Another golfer, *Admire*, had a total score of:

||/|| ||/|| ||/|| ||/|| |||

on five holes. Whose score was better (Matthew or Admire)?
(b) Another golfer, *Chipo*, had a total of 26 on <u>six</u> holes.
Write her score in talley notation.

Problem 4 refers to data in Problem 3.
1.4. (a) Find the average number of strokes per hole for each of the golfers.
(b) Explain what difficulty you might have in writing the average in talley notation.
(c) Which of the three golfers had the best (the lowest) score

1.5 Construct a table using tallies that represent the following event as it occurred. We were riding along in a Game Park near Mutare and we saw a zebra, a giraffe, an antelope, another giraffe, two more zebras, an elephant, another giraffe, five more antelope, another zebra, two more antelope, another elephant, a hippo, a rhino, and another antelope.

CHAPTER 2. TABLES

Reading and Using Tables

A *table* is an arrangement of data (numbers) into *rows* and *columns*. Tables help you understand the relationships between various quantities. It is important to clearly keep straight which are the rows and which are the columns. And you need to know which row is first row, second row, ... and which column is the first column, second column, etc.

Every table has a top row (horizontal list of numbers) and a left-most column (vertical list of numbers). The *top row* of numbers (not counting the heading) is the *first row*, the row below that is the second row, the one below that is the third row, etc. The *left most column* of numbers is the *first column* the one to right of it is the second column. etc.

A TABLE WITH FOUR ROWS
AND SIX COLUMNS
(The headings "Row.. " and "Col.." do not, themselves, count as rows or columns)

	Col 1 ↓	Col 2 ↓	Col 3 ↓	Col 4 ↓	Col 5 ↓	Col 6 ↓
Row 1 →	2	3	5	7	11	23
Row 2 →	4	6	10	14	22	29
Row 3 →	13	17	19	23	29	31
Row 4 →	26	34	38	46	58	37

Table 2.1

The heading of the first row is "Row 1 → " .The numbers in the first row are:
2; 3; 5; 7; 11; 23.
The heading of the first column is "Col 1 ↓ ". The numbers in the first column are:
2; 4; 13; 26.
A number in some given row or column is the *entry* data for that row and column. For example, the number in the second row, fourth column in the above table is 14; so the (second row, fourth column) entry is 14. The number 23 is entered twice. In (row 1, column 6) and in (row 3, column 4).

Exercises

2.1. (a) Using the table above write down all of the numbers of the third row.
(b) Write down all the numbers of the fourth column.
(c) Write down the (row 4, column 3) entry and the (row 3 column 4) entry.
(d) Explain why you cannot write down a (row 5, column 4) entry.
(e) Write down the two locations of the number 29.

2.2 (a) Fill in the missing numbers in row 2 for the following table:

x	0	1	2	3	4	5
x^2	0	1		9		

Table 2.2

(b) How is row 2 related to row 1?
(c) Write down the first column in Table 2.2

2.3 Read data from the following table to answer the questions below.

TABLE SHOWING THE RELATION BETWEEN THE LENGTH OF A PENDULUM AND THE NUMBER OF SWINGS (DUE TO GRAVITY) PER MINUTE

Pendulum	Length (cm)	No of swings/minute
A	20	62
B	30	54
C	40	48
D	50	44
E	60	39
F	80	31
G	100	28

Table 2.3

(a) Use the table to determine whether increasing the length of the pendulum *increases* or *decreases* the number of swings per minute.
(b) From the table, which pendulum has the shortest length?
(c) Which pendulum has the fewest number of swings per minute?
(d) How many swings does Pendulum D make in one minute?
(e) Calculate the time (in seconds) it takes Pendulum A to make one swing.
(f) Calculate the time (in seconds) it takes Pendulum E to make one swing.
(g) Use the table to estimate how many swings per minute would there be for a pendulum of 90 cm.

2.4 When growing bacteria culture, you measure the population size by the area of the *substrate* or medium covered by the bacteria. If you compute the area at regular time intervals (every hour) you can keep a record, in table form, of the population growth. Suppose you have acquired the following data in your notebook. Consider the experimental data to have begun with the 1:00 measurement. Don't try to assign a value to the population size at 12:00.

- ♦ Started the bacteria colony at 12:00 noon.
- ♦ Measured the area of the bacteria culture at 1:00 PM, and it is 10 cm^2.
- ♦ Measured the area of the bacteria at 2:00 PM; it is now 15 cm^2.
- ♦ The culture size at 3:00 PM is 22 cm^2.
- ♦ The culture now measures 33 cm^2 $at\ 4$:00 PM.
- ♦ 50 cm^2 at 5 Oclock.
- ♦ Now the bacteria covers 75 cm^2 and it's only 6:00 PM, I'm getting scared!
- ♦ At 7:00 PM the culture is 108 cm^2.
- ♦ I forgot to measure it at 8:00, but now at 9:00 it is 243 cm^2. I am putting in the inhibitor and shutting off the population growth so I can go home.

(a) Construct a table with two columns the first headed with "TIME" and the second with "AREA". Label the Table as "BACTERIA GROWTH OVER AN INTERVAL OF NINE HOURS." (b) Use your table to make an estimate of the culture size at 8:00 PM.
(c) Estimate the population size at 3:30 PM.

Tables of data are conveniently recorded in vertical form such as Table 1, from Problem 1.2:

Hole N^o	N^o of strokes
1	||/|
2	||||
3	|||/ |
4	||/| ||
5	|||

Table 2.4

But to save space on the printed page, most texts write them in horizontal form like this:

Hole N^o	1	2	3	4	5
N^o strokes	|||/	||||	|||/ |	|||/ ||	|||

Table 2.5

Notice that when you convert from a table from the vertical form to the horizontal form, you must make the *first column* become the *first row*, and the second column become the second row, etc. You follow a similar rule (in reverse) when converting from a horizontal table to a vertical table.

2.5. (a) Re-write the pendulum table in the following format:

RELATION BETWEEN A PENDULUM LENGTH
AND THE NUMBER OF SWINGS PER MINUTE

Pendulum	A	B	C	D	E	F	G
Length (cm)							
N^o of swings/min.							

Table 2.6

(b) Re-write the bacteria growth table into the horizontal format (Make sure you write the data in the first column as the first row of the converted table.

(c) Re-write the following table in the vertical format.

TABLE SHOWING THE RELATION BETWEEN CARBON ATOMS AND
HYDROGEN ATOMS IN THE HOMOLOGOUS SERIES, ALKANE

C (atoms)	1	2	3	4	5	...	n
H (atoms)	4	6	8	10	12	...	$2n+2$

Table 2.7

(Remember to make the first row become the first column in the new table)

CHAPTER 3. GRAPHS

A graph is another way to express the data given in a table. You can plot the data from a table onto a graph in the xy-coordinate system. (The x-axis is the ***number line*** or the ***first*** axis, and the y-axis is the ***second*** number line). In a vertical table, all the numbers in the first column correspond to numbers on the first axis, the x-axis and all the numbers in the second column correspond to numbers on the second axis, the y-axis.

Histogram

There are several types of graphs. One is called the ***histogram***, which is a set of parallel vertical line segments place so that their ends are located at the numbers in the first column and their lengths are equal to the corresponding numbers in second column.

Example:

Make a histogram of the table:

x	y
1	3
2	6
$3,5$	2
5	-4
7	1
10	5

Table 3.1

This is done in two steps.

Step 1. Mark the first axis (the x-axis) with the numbers in the first column.

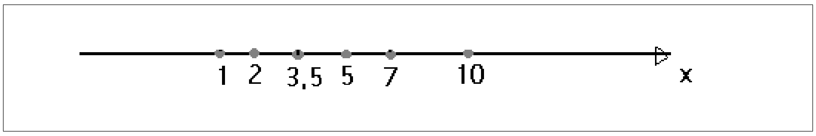

Figure 3.1 Plot of the numbers from the first column.

Step 2. At each of these numbers, construct a vertical line segment whose length is the corresponding number in the second column.

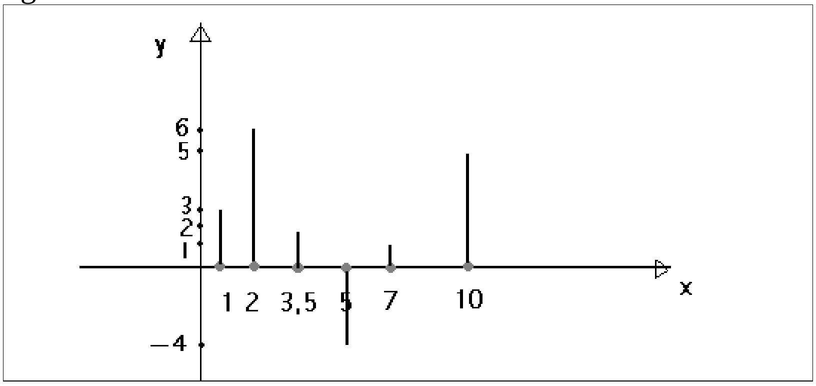

Figure 3.2 Histogram for Table 3.1

The lengths of vertical segments are the numbers in the second column.

In the above table, notice that the -4 is the y value that corresponds to $x = 5$; therefore, in the histogram, the segment constructed at $x = 5$ has a length of -4 which is drawn below the x-axis.

Exercises

3.1 (a) Draw a histogram for the data from the following table:

x	y
1	4
2	6
3	8
4	5
5	4
6	2

Table 3.2

(b) Make a ***bar graph*** for this table by drawing narrow vertical rectangles in place of the vertical line segments you used in the histogram.

Bar Graphs

You can represent the data from a table by drawing a bar graph, which is just like the histogram, only you use a vertical rectangle in place of the vertical line segment. It is also a two-step process, but in the second step, at each number of the first column, you construct a vertical rectangle whose height is the corresponding number in the second column.

Example, starting with the histogram in Figure 3.2, and enclosing each vertical line segment in a rectangle, we get:

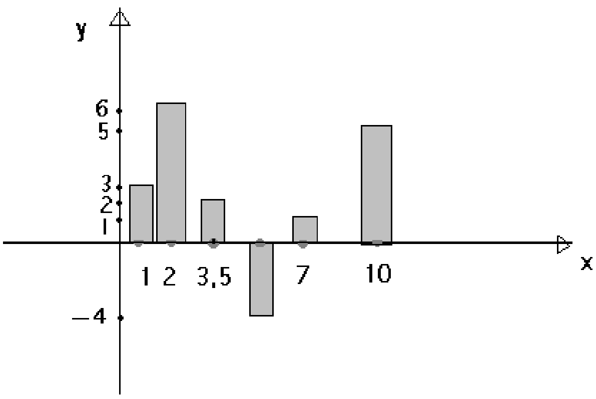

Figure 3.3 Bar graph for the histogram in Figure 3.2

3.2. Draw a bar graph for the data in Table 3.3, below. Make the bars wide enough to touch each other.

x	1	2	3	4	5	6	7
y	2	3	2	4	1	6	9

Table 3.3

Line Graphs:

A line graph can be obtained by starting with a histogram and connecting the tops of the positive vertical line segments and the bottoms of the negative ones, either by straight lines or by smooth curves. Here are two examples of line graphs using the data in Table 3.1

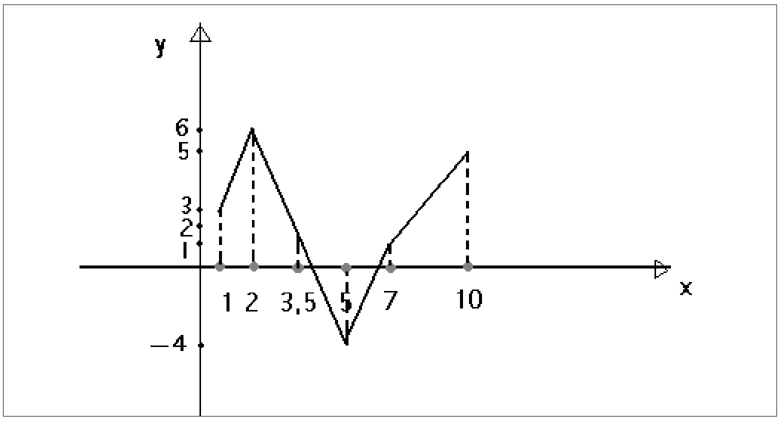

Figure 3.4. A line graph for Table 3.1 using straight line segments

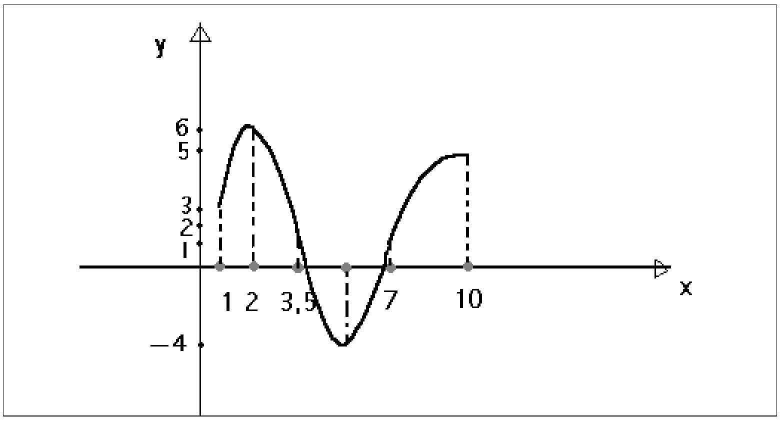

Figure 3.5 A line graph for Table 3.1 using a smooth curve

3.3. (a) Plot the data from the following table onto the xy-axes and draw a smooth curve connecting the points.

x	0	1	2	3	4	5
y	0	4	6	6	4	0

Table 3.4

(b) Estimate the value of y when $x = 2\frac{1}{2}$
(c) Estimate the value of y when $x = 6$
(d) Estimate the value of x at which the graph reaches its highest point.
(e) Estimate two values of x that make $y = 3$.
(f) Show that the equation $y = x(5 - x)$ fits every pair of numbers in the table.

(This means that if you select any x in the first row of the table then the corresponding y value will be <u>that</u> x times the quantity 5 decreased by <u>that</u> x.)

3.4 In a biology experiment, the water levels in a capillary tube at three minute intervals, are as depicted in the diagrams below:

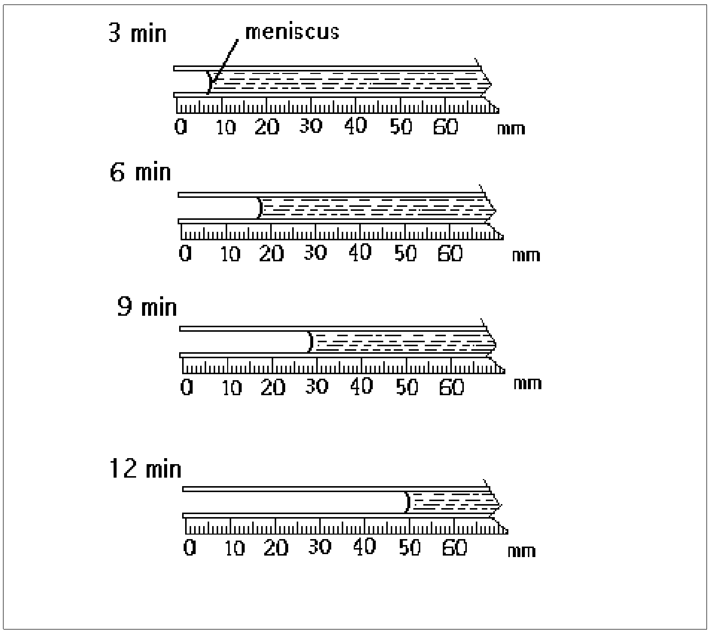

Figure 3.6.
EVAPORATION AS MEASURED IN A CAPILLARY TUBE

The meniscus is the in the center of the tube and is the point at which the water level is measured. At time 0, the meniscus was set at 0 in the capillary tube.

(a) Use the readings in Figure 3.6 to complete the following table.

Time (minutes)	0	3	6	9	12
Distance moved by the meniscus	0				

(b) Construct a graph of the results in your table. Mark a scale on the two axes so as to use the entire grid in your graph. (Clearly mark the two axes.)

(c) How fast (in mm/minute) did the meniscus move during the first three minutes? How fast did it move during the last three minutes? Did it have a uniform velocity?, Explain.

3.5 Look at the graph in Figure 3.7, below and answer the following questions (assume time is in seconds):

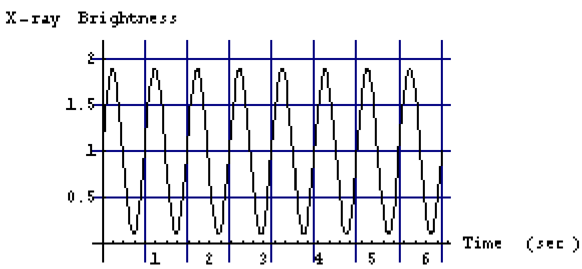

Figure 3.7
FLUCTUATION OF A PULSAR OVER TIME

(a) What is the pulsar's x-ray brightness at 0.5 seconds?
(b) At what times are the x-rays at their maximum?
(c) At what times are they minimum?
(d) What is the brightness number for the pulsar at 3.4 seconds?

3.6 Use data from the following graph to complete the table below.

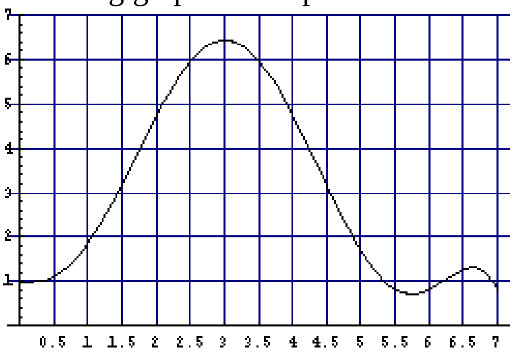

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Figure 3.8

(a) Table for the data in Figure 3.8

x	0	1	2	3	4	5	6	7
y	1							

(b) Re- write the table in Part (a) in vertical format.

3.7. Refer to Problem Number 2.4 (The bacteria growth problem). In your table for that problem, the first column should be headed "Time" and the second column "Area". Use your table to construct a line graph called "Bacteria Growth Over Time", by plotting the points, and drawing a smooth curve through them. Label the axes as "Time" and "Area". Use you graph to estimate the area of the bacterial colony at 1:30 PM

3.8 Refer to Problem Number 2.3, and Table 2.3. Use the "Length of the Pendulum" as the data on the x-axis and "Number of Swings per Minute" as data on the y-axis. Plot the points from the table and draw a smooth curve through them. Use your graph to estimate the number of swings per minute of a pendulum that is 55 cm in length. From the graph what do you think happens to the number of swings per minute if the length of the pendulum is made longer and longer?

NON-PARAMETRIC BAR GRAPHS

Sometimes we may wish to draw a bar graph for data that is not based upon two columns of numbers. A table may have one or more of its columns representing some category, some condition, some type of chemical reaction, some type of animal, etc.

Example: Consider the following table constructed from the data in Problem 1.5

Type of Animal	Number
Antelope	9
Elephant	2
Giraffe	3
Hippo	1
Rhino	1
Zebra	4

TABLE 3.5 TYPES OF ANIMALS AND NUMBER OBSERVED

Here, we do not make the first column correspond to a set of numbers on the x-axis, but rather to a list of the types of animals , but we still use numbers vertically (of the y-axis) to stand for the number of animals seen. The entries in Column 1 are called *non-parametric* because they are not numbers. We will layout the names of the animals alphabetically then, at each name, construct vertical bars whose length will be the corresponding number in the second column.

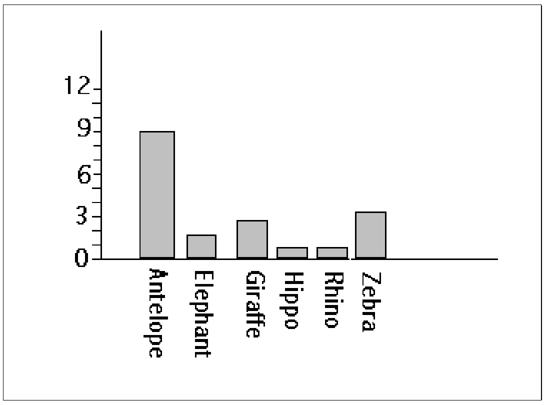


Figure 3.9 Bar graph for Table 3.5

3.9 Construct a non-parametric table and draw a bar graph for the following data.
In our back yard one afternoon we saw 14 Crows, 8 Robins, 2 Woodpeckers, 3 Sun birds, 1 Purple Crested Lourie, 2 Herons, 5 Wagtails, and no Hoopoe.

3.10 The table below shows the effect on yields of a maize crop under different growing conditions. Draw a bar graph.

Soil Conditions	**Yield (Kg/hectare)**
nothing added	920
nitrogen added	1430
nitrogen and phosphorus added	1516
cattle manure added	2137

CHAPTER 4. EQUATIONS

Meanings

Equations are sentences. But they are written in mathematical language. In mathematical sentences we use the $equal\, sign$ ($=$) to stand for words like **is** or **are** or **were** or **will be** or **is the same as** or **equal to**.

Examples:

English sentence	Equation	Explanation
"Equations are Sentences"	$E = S$	E stands for equations and S stands for sentences
"The price of this orange is $5"	$x = 5$	x stands for the price of the orange
The number of monkeys on my roof this morning was 12	$m = 12$	m stands for the number of monkeys on the roof
f goals in the 1st half plus $s$goals in the 2nd will be g goals in the game	$f + s = g$	f stands for the number of goals in the 1st half, etc.
"My money is worth eight times your money"	$m = 8y$	m is one rand and y is one Zim dollar
"2 bags of tomatoes plus 3 bags of tomatoes is worth $100"	$2b + 3b = 100$	b stands for the price of one bag of tomatoes.
Hydrochloric Acid and Sodium Hydroxide make salt and water	$HCl + NaOH \rightarrow NaCl + H_20$	HCl stands for Hydrochloric acid, etc.

Types of Equations:
There are three types of equations:

♥ $Those\, that\, are\;true\; for\; all\, numbers$ (Example: $3xy + xy = 4xy$)

♣ $Those\, that\, are\, true\; for\; some\, numbers\, but\, not\; for\; all$ (Ex. $3x + 4x = 20$)

N $Those\, that\, are\, never\, true\; for\; any\, number$ (Example: $x + 5 = x$)

Identities

The equations that are <u>always true </u>(true for any number that you substitute in for the variables) are called *identities*.
Examples:
(a) $a \times (b + c) = a \times b + a \times c$ is true no matter what values you substitute in for the variables, a, b, c.
(b) $7x + 3x + 5 = 10x + 5$ is true no matter what value you use for x..
Try any number for x you will see that the equation is true.

(c) Similarly you can substitute anything for x and anything for y into the equation: $7x \times 3y \times 5x = 105x^2y$ and it will be true for those values.

Conditional equations

The equations that are conditionally true (that is, true for some values of the variables but not for others) are called *conditional equations*.
Examples:
(a) The equation $7x + 3x + 5 = 90$ is not true for $x = \frac{1}{2}$. And it is not true for $x = 3$.
You can try these values and you will get the false statements $10 = 90$ and $35 = 90$, respectively. The only value of x that you can use to make the equation true is $x = 8\frac{1}{2}$. Try it, substitute in $8\frac{1}{2}$ for x.

$$7 \times (8\tfrac{1}{2}) + 3 \times (8\tfrac{1}{2}) + 5 = 90$$
$$59\tfrac{1}{2} + 25\tfrac{1}{2} + 5 = 90 \text{ or } 90 = 90.$$

(b) The equation $x^2 - 5x + 6 = 0$ is a conditional equation. It is not true for any value of x except $x = 3$ or $x = 2$. Try $x = 3$, you get $9 - 15 + 6 = 0$ or $0 = 0$. Similarly $x = 2$ gives you $4 - 10 + 6 = 0$ or $0 = 0$. But any other value substituted in for x, will give you a false statement.

Contradictions

Some equations are not true for any choice of the variables. These are called *contradictions* or inconsistencies.

Example: There is no value of x that makes $x + 1 = x$. Nor is there any number x, such that $\frac{x}{x} = -1$

Exercises

4.1 Identify which of the following equations are identities, conditional equations or contradictions.

(a) $x + (-x) = 0$
(b) $x = 3 - 2x$
(c) $y = mx + b$
(d) $\frac{1}{x} + \frac{1}{y} = \frac{2}{x+y}$
(e) $3xy + 7 = 7xy + 3$
(f) $x^2y + x^2y = 2x^2y$
(g) $x(y - z) = xy - xz$
(h) $x + 10 = 2x$
(i) $x + 10 = x$
(j) $x^2y + x^2y = x^4y$

4.2 (a) Is the equation: $x^2u \times x^2u = x^4u^2$ an identity or a conditional equation?
(b) Is the equation : $x^2u + x^2u = x^4u$ an identity or a conditional equation?
(c) If either (a) or (b) is a conditional equation solve it for x.

4.3 Show that the equation: $\frac{1}{x} + \frac{1}{y} = \frac{2}{x+y}$ is <u>not</u> an identity by finding values of x and y that make it false. (Note: Neither x nor y can be zero, because you may not divide by zero.)

4.4 Show that $\frac{1}{x} + \frac{1}{y} = \frac{2}{x+y}$ is actually a contradiction; that is, <u>there are no values of</u> x and y that can make it true.

Most equations used in the applications to Physics, Biology, Business and other various fields are <u>conditional equations</u>, not identities (and not contradictions). The value of one variable depends on the values assigned to another variable. For example, if you travel a certain distance, d in a certain amount of time t, then your average speed for that trip is $distance$ divided by $time$ or:

$s = d/t$

Where s is the average speed for the trip, d is the distance traveled and t is the time it took. This equation is a **formula** or a **definition** for speed. It is not an identity, because you can find three numbers to substitute in that would make the equation false. For example, if you substitute in $d = 100km$ and $t = 2\,hours$, and $s = 3km/hour$, You will get: $3km/hour = 50km/hour$, which is clearly false. Although it is not an identity, it is not a contradiction because there are substitutions for the variables that make it true.

This type of equation is not only an English sentence in mathematical symbols, but it is a law, definition, or formula. It has an additional meaning that you *interpret* in some descriptive way.

4.5 Velocity is defined as displacement ("directed distance", or motion in a given direction) over a given time. $v = x/t,$ Where x is a directed distance, and t is time. (Velocity is a vector quantity; it has direction and magnitude.)
(a) If a body is displaced upward, away from the earth a distance of 250 km in 22 seconds, find its velocity. (Assume upward displacement is positive)
(b) If a body is propelled downward, towards the earth, (negative displacement) for a distance of 100 km in 14 seconds, find its velocity.
(c) If a body has a velocity, $v = -42km/sec$, for 6 seconds, find its displacement.
(d) If a body has a velocity, $v = 42km/sec,$ for 6 seconds, find its displacement

4.6 Acceleration is defined as the change (increase or decrease) in the velocity of an object over time.
(a) An experiment is done in which an object is moving at a velocity $v_1 = 17m/s$ at the beginning of the experiment and is moving at velocity $v_2 = 22m/s$ at the end, and the experiment took two seconds, what was the average acceleration over that time?
(b) If during the first ten seconds, a rocket goes from a velocity of $v_1 = 0$ to a velocity of $v_2 = 500km$/h, find the average acceleration in m/sec^2 during those 10 seconds.

4.7 A computer in a department store has the formula:
$V = a_1x_1 + a_2x_2 + a_3x_3$
for the Kitchen Department.

(a) If a_1 is the price of 4-cup coffeemakers and x_1 is the number of 4-cup coffeemakers in stock, then what does the product a_1x_1 represent?
(b) If a_2 is the price of Champaign glasses and x_2 is the number of Champaign glasses in stock, what does a_2x_2 represent?
(c) If a_3 is the price of a set of silverware and x_3 is the number of sets of silverware in stock. Interpret what the formula $V = a_1x_1 + a_2x_2 + a_3x_3$ represents?

4.8 A game park calculates its revenue on the following basis,
$p_1 =$ admission charged to resident adults (16 years or older)
$p_2 =$ admission charged to resident school children (under 16 years)
$p_3 =$ admission charged to foreign tourist adults
$p_4 =$ admission charged to foreign tourist children.

(a) Let $x_1,\ x_2,\ x_3, x_4$ be the <u>number</u> of visiting resident adults, resident children, foreign tourist adults and foreign tourist children, respectively. Write an equation that

gives you the revenue, say R in terms of these prices (the p_i's) and the number of visitor (the x_i's)

(b) The prices, in Zim Dollars, are as follows: $p_1 = 30,\ p_2 = 10,\ p_3 = 1000,$ $p_4 = 250.$
The gate records for one week show that the number of resident adults admitted was 280, the number of resident children was 450, the number of foreign tourist adults was 64 and the number of foreign tourist children was 30, Find the revenue that should be reported.

(c) If in another week, $x_2 = 13,\ x_3 = 800, x_4 = 210,$ revenue was $R =$ \$Z $864\,000$, find x_1, the number of resident adult visitors.

Solving Equations.

You can sometimes find the values of the variable that makes a conditional equation true. This is called "solving" the conditional equation. You can also show whether or not a number makes the equation true or false. This is called "checking" a possible solution. One method for solving an equation is called "guess and check" You guess at a value that might make the equation true, then you check to see whether or not it actually does so. Guess and check is an inefficient method, so let us learn some better ways to do it.

To solve a conditional equation you try to perform equal operations on both sides of the equation, so that at every step the equation remains conditionally true. Usually you add, subtract, multiply or divide both sides of the equation by the same "thing" (expression, number, power, etc.)

EXAMPLE Solve the equation $3x + 7 = 19$ for x.

1. Start with the original equation: $3x + 7 = 19$
2. Subtract 7 from both sides: $(3x + 7) - 7 = 19 - 7$
3. Simplify both sides $3x + 0 = 12$, or $3x = 12$
4. Divide both sides by 3 $3x/3 = 12/3$
5 Simplify both sides $x = 4$

In the original equation, 7 was added to $3x$. We opposed the ***added*** 7 by ***subtracting*** 7 from both sides. Then in the equation $3x = 12$, the variable x was ***multiplied*** by 3, we opposed this by ***dividing*** both sides by 3.

Using this method of undoing <u>the same operation</u> on both sides will do two things:
1. Keep the equation balanced so that it will be true under the same conditions as the original equation.
2. The equation will be "reduced" (or simplified) at each step because the original operations are opposed until the variable appears by itself on one side of the equation.

Example:
Solve the equation $s = \frac{d}{t}$ for d.
1. Start with the original equation: $s = \frac{d}{t}$

2. Do something to both sides, but what is that something?, add? subtract? multiply?, divide? Well here you are trying to solve for d and you notice that d is divided by t, so the something you do it to multiply both sides by t.

$$s \times t = \frac{d}{t} \times t$$

3. Simplify: $s \times t = d$

So we have solved for d, it is $s \times t$.

Exercises

4.9 (a) Solve the equation $10x + 13 = 22$, for x.
(b) Solve the equation $\frac{A}{B} = C$, for A.
(c) Solve the equation $\frac{A}{B} = C$, for B

4.10 (a) A cube with side 5 mm has a total surface are of how many squared mm?
(b) This same cube has a volume of how many cubic mm?
(c) Find the surface to volume ratio of the cube with side 5 mm.

4.11 One micrometer (1μ) is one- one thousands of a millimeter. Written as equation this says: $1\mu = mm/1000$.
(a) Express the length $1\ mm$ in terms of μ.
(b) Express the area $1\ mm^2$ in terms of μ^2.
(c) Express the volume $1\ mm^3$ in terms of μ^3.
(d) A cell has a volume of $0.000\,34\ mm^3$, express the cell's volume in μ^3

4.12 A cell is known to have a volume of 11 900 μ^3 and a surface area of 3000 μ^2,use the equation: SVR (Surface to Volume Ratio) $= \frac{Surface\ Area}{Volume}$ to find the SVR in terms of μ^2/μ^3, (or μ^{-1})

4.13 The surface to volume ratio (SVR) of a cell was found to be $500 : 1\,mm^2/mm^3$. Use the equation $SVR = \frac{Surface\,Area}{Volume}$ to find the surface area if the volume of the cell was $4.6 \times 10^{-5} mm^3$. First find the area in mm^2, then in μ^2.

Relations of Equations to Graphs.

The equations that are used to represent graphs usually fall into one of the following five major categories.

1. Equations of a straight line. Example: $y = -3x + 10$
2. Equations of a parabola. Example: $y = \frac{1}{2}x^2 - 13x + 23$
3. Equations of an exponential function
   Growth curves, Example: $y = 2.3 \times 3^x$
   Decay curves, Example: $y = 10 \times 3^{-x}$
4. Equations of a logarithmic curve. Example $y = log(2x)$
5. Equations of periodic functions. Example $y = 1.4 \times sin(2.1x)$

The student could save a lot of time and obtain more accurate results if she could recognize, just by looking at the equation, what kind of graph it is going to have.

4.14. (a) Determine the kind of graph that is represented by the equation: $y = 2 \times e^{1.3x}$.

(b) Determine the kind of graph that is represented by the equation:

$y = 2 + Sin(2x + 1)$

(c) Determine the kind of graph that is represented by the equation $y = 2^{-.5x}$

(d) Determine the kind of graph represented by the equation $y = (x - 3)^2$

(e) Determine the kind of graph represented by the equation $y = log_5(x)$

A NOTE ON GRAPHS AND THEIR EQUATIONS

If we know an equation such as:

$$y = 2x^2 - 3\,x + 5$$

we can usually draw the graph by plotting the points $(x;y)$ that we get by substitution of values for x. Thus, for example, if we let $x = 1$, we will get $y = 2 - 3 + 5 = 4$. So we plot $(1; 4)$, then if we let $x = 2$, we get $y = 8 - 6 + 5 = 7$. So we plot $(2; 7)$ etc. Thus for any suitable value $x = a$, we get $y = 2a^2 - 3a + 5$ and we then plot the point $(a; 2a^2 - 3a + 5)$. In fact, we say that the graph of the equation is the set of all points whose coordinates are: $(x; 2x^2 - 3x + 5)$.

Similarly, if we already have a graph whose coordinates we know, then we can sometimes get the ***equation*** from the coordinates. For example, If all of the coordinates of a graph G are of the form $(x; \frac{2}{x^2+4} + 17x - 3)$, then we know that the equation of the graph is

$$y = \frac{2}{x^2 + 4} + 17x - 3.$$

Most of the time we might have a graph that we obtained from experimental data and we do not know the coordinates in a general form. But we can try to fit an equation to the graph by trying to determine whether or not there is some (approximate) relation between the x-coordinate and the y- coordinate of all of its points.

4.15 (a) Given that a graph A has coordinates $(x; \frac{(x-1)}{(x-2)(x-3)})$, write its equation.

(b) Given that a graph B has equation $y = \frac{2}{x^2+4} + 17x - 3$, write the coordinates of the point P whose x-$coordinate$ is p.

CHAPTER 5.
VARIABLES, CONSTANTS AND FUNCTIONS

In science and in maths, a *variable quantity* is something that may represent a measurement that can vary according to when or where the measurement is taken. For example, when the ZESA worker come to read your electricity meter, the number that he or she reads varies according to how much electricity you used since the last time the meter was read. A *constant quantity* is something that stays the same during some time or over some distance. For example, your house number does not vary between visits by the ZESA meter reader, it is a constant quantity.

Unfortunately, in mathematics (and in science) when we talk about a variable quantity, we often drop the noun "quantity" and simply use the adjective "variable" to stand for the variable quantity. So when we say "x is a variable", we really mean that x is a number that stands for a measurement that varies in some way. Similarly, when we say "B is a constant" we mean that B is a number that does not vary within the context of the particular discussion.

But, constants may also vary in another sense. As you walk down the street, the house numbers are varying, but *on each house the number is a constant for that house*.

Examples: The equation $y = 3x$ +1 has a graph that is a straight line (with a constant slope of 3 and passing through the point (0;1). As the value of x varies, then y will also vary at a rate that is 3 times as fast as x. The equation $y = 4.7x \ - 2$ is a straight line with a constant slope 4.7 and passing through the point $(0; \ -2)$. The general equation $y = mx + b$ is a straight line with constant slope m and passing through the (constant) point (0;b).

Sometimes the "variable" is used as an "unknown" (that is an unknown quantity). Such as in the conditional equation: $3x + 1 = 0$. Here the equation is not true for just any old variable x, but only for the value $x = \ -\frac{1}{3}$, a constant.

In general, letters of the alphabet stand for numbers in mathematical expressions and whether the quantities represented are constants or variables depends upon the context of the problem.

FUNCTIONS

An equation that relates variables and constants may define a *function* when one of the variables is dependent upon the values of one or more of the other variables.

A function is sometimes called a "rule" that lets you relate two variables. In this sense a function could be a table, a graph, or just a collection of coordinates $(x; y)$ that uniquely identifies a value of y with a given value of x.

Suppose that $(x; y)$ is a pair of coordinates for a given function F. Then the value of y that corresponds to the given x is sometimes denoted as $f(x)$, to be read as "f of x" or "f at x" or ″the value of f when the first coordinate is x")

EXAMPLE: Let G be the function defined by the following table of values.

x	0	0,5	1	1,5	2	2,5	3	3,5	4	4,5	5
y	0	2,25	4	5,25	6	6,25	6	5,25	4	2,25	0

Table 13

Let $g(x)$ denote the value of y when the first coordinate is x. Then $g(1) = 4$ and $g(1,5) = 5,25$ and $g(5) = 0$; etc. In general this table is obtained from the equation: $y = g(x)$, where $g(x) = x(5 - x)$ for any number x.

In science, the use of a function such as $g(x) = x(5 - x)$ would not be so sterile as the above example illustrates, because it would have some practical interpretation. For example in biology, the function $g(x)$ could be the growth rate (in grammes per hour) of a population of bacteria in a petri dish that could hold only 5 grammes of bacteria and x could be the weight of the bacteria. As the population grew from 0 gm to 2,5 gms (half filling the dish) the growth rate would be very high (at 6,25 gms/hour), but as the dish became over crowded, the growth rate would decline back to zero.

Problems:

5.1 In biology, the weight of a fish is proportional to the cube of its length. $W \propto l^3$. Suppose that we have several sizes of the same kind of fish; they have varying lengths which we will represent by the variable l. We will assume that there is a constant k, such that the weight of the fish is that constant times the cube of the length of the fish. That is:

$$W(l) = k\, l^3$$

(a) If one fish is twice as long as another, how many times heavier is the large fish?

(b) If a 19 cm fish weighs 892 grams, find the value of the constant k to 3 significant figures.

(c) Using the constant you found in part (b) compute $W(14)$, $W(7)$, $W(21)$, $W(28)$ in grams.

5.2 Suppose a box of weight W Kgm is sitting on the floor and the box covers an area of A meter^2. The pressure ρ (in Kg/m^2) exerted by the box is said to be proportional to the weight divided by the area. This is sometimes written as $\rho \propto$ W/A. We can change this <u>proportionality</u> into an <u>equality</u> by simply saying there is some constant k such that the pressure is equal to k times the weight divided by the area:

$$\rho = \frac{kW}{A}$$

We will write the pressure ρ as a function of two variables W and A. $\rho(W, A) = \frac{kW}{A}$

(a) If the mass of the box is 7 kg (so the weight is 70 newton's) and the area of the bottom of the box is 900 cm^2 calculate the pressure under the box in N per m^2.
(Remember 1 $cm^2 = \frac{1}{10000} m^2$)

(b) If 1 Pascal (Pa) is the pressure of 1 Newton per square meter, calculate the pressure under the box in Pa. In kilopascals. (a Kilopascal is 1000 pascals)

Constants, variables and formulas in computer science:
Variables and constants in computer science have a further complication. When the value of a variable or constant or any other number is provided by the computer it is always an *approximation*. When the computer prints the number 3, for example, it might be storing it as the number 3 + 0,0000001257880915 or as 3 - 0,0000001257880915 or any number in between. If you keep dividing this number in half over and over and over again, the result becomes closer to zero until the number is below the threshold of the computer's ability to recognize that the number is not zero and it will print an error message indicating "underflow". We even have the case where the computer cannot handle ordinary "human-versions" of a simple formula such as the the solution to the general quadratic equation:

$$ax^2 + bx + c = 0$$

This equation has real solutions if the three constants a, b, c satisfy the inequality: $b^2 - 4ac \geq 0$. Otherwise the solutions are *complex* (using the number $i = \sqrt{-1}$). The real roots to the equation has the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

as its human-version.
In computer science, however, the solution $x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a}$ can cause a computational problem when a is very small, solution becomes dangerously close to an attempt to compute and indeterminate form $\frac{0}{0}$.(See what happens if you let $a = 0$ in this solution).
To get around this problem computer scientists use an alternate formula for this solution. It is

$$x_1 = \frac{2c}{-b - \sqrt{b^2 - 4ac}}$$

Now, there is no problem when a is small. In fact if $a = 0$ you get the exact answer $x_1 = -\frac{c}{b}$ to the equation $0x^2 + bx + c = 0$.

5.3 Show that, algebraically

$$\frac{-b+\sqrt{b^2-4ac}}{2a} = \frac{2c}{-b-\sqrt{b^2-4ac}}$$

even though, computational we cannot compute the left hand side for $a \approx 0$.
HINT: Multiply the numerator and denominator of the left hand side by $-b - \sqrt{b^2 - 4ac}$.

Variables and Constants in Graphs

When we want to draw the graph of an equation we should first look at the way in which variable and constants appear in the equation. This will give us some clues for drawing the graphs.

Example: In the equation $y = 3 + \frac{1}{(x-2)}$, the 3 tells us that this graph will always be a constant 3 units above (and parallel to) the graph of the equation $y = \frac{1}{(x-2)}$. The variable is x and it can be anything except 2, because if $x = 2$ the equation will say that 1 is divided by $2 - 2$, which is to say that 1 is divided by 0, which is not allowed. Therefore the constant 2 is a forbidden number in this equation. If the graph is to have a value at $x = 2$ then it must be given as a separate piece of information. If we pick a value for the variable x "close to" 2, then the fraction will be a large positive or a large negative number. (If $x = 2.01$, then $y = \frac{1}{(x-2)} = \frac{1}{.01} = 100$, If $x = 1.999$, then $y = y = \frac{1}{(x-2)} = \frac{1}{-.001} = -1000.)$ This means that y increases indefinitely (to $+\infty$) or decreases indefinitely (to $-\infty$) as x gets close to 2. Thus, the graph approaches a vertical line as x approaches 2.

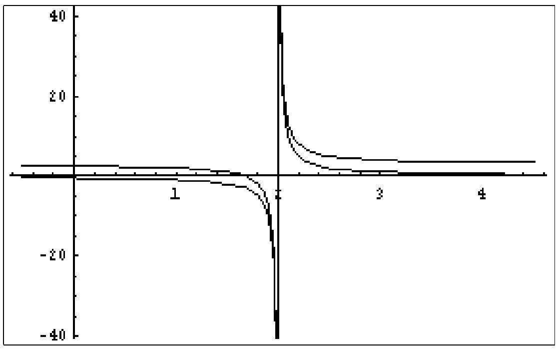
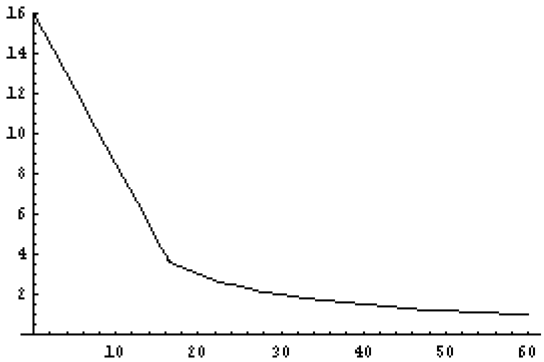


FIGURE
GRAPHS OF $y = \frac{1}{(x-2)}$ AND $y = 3 + \frac{1}{(x-2)}$

Problems

5.4 Analyze the equation $y = -4 + \frac{\frac{1}{2}}{(x-3)}$ in terms of the constants and sketch the graph.

5.5 In a physics experiment, a spring was compressed by adding weights. When the weights were less than 16 N, the spring's length decreases linearly (with a constant negative gradient). When more than 16 N were added the compression began to have a varying gradient. The result of the data is depicted in the following graph.



An equation was proposed for this graph:

$$L = 16 - \frac{3}{4}N \text{ if } 0 < N < 16.5$$
$$L = \frac{60}{N} \text{ if } 16.5 < N < 60$$

(a) Find $L(16)$, Find $L(17)$
(b) Are these equations reasonable for this experiment?
(c) What physical law describes the compression of springs?

CHAPTER 6. MEASUREMENTS

Practical Measurements--Use of measuring tools

Say that we suspend a spring from a horizontal bar and hang a weight on it. We want to measure the length of the spring before and after we stretched it by hanging the weight. We will try to observe whether or not there is a relationship between the amount of weight and the amount of stretch. This experiment involves the measurement of a distance (or length) in centimeters and a weight in grammes. Both of these measurements require a measuring tool, a ruler for the distance and balance for the weight.

In chemistry you might use a measuring cylinder calibrated in ml to measure the volume of a given solvent.

In mathematics you may be required to construct a triangle whose three sides are: 3 cm, 4 cm, and 5 cm. Then you would be required to measure the angle between the two shortest sides with a protractor.

In all three cases we are obtaining approximations to the quantities being measured. These practical measurements are limited by the accuracy of the tools you are using to make the measurements. For example, if the finest scale on a ruler is mm, we will not be able to measure anything to an error less than $\pm$ 1 mm.

Ideal Measurements--Use of Theorems and Laws

On the other hand, all fields of science and mathematics have *theoretical quantities* in which *perfect measurements* would produce these theoretical quantities with 0% error (100% accuracy). For example, in maths we say that in an equilateral triangle, every angle has an exact measurement of 60°, and we accept the proof of this without trying to measure the angles.

In discussing general mathematical and scientific concepts we usually proceed as if all our measurements are ideal. So, for example, when we calculate the ratio of the *surface area* of a spherical cell to its *volume*, we assume that the diameter of the cell is given as some number d measured to 100% accuracy, and that the volume is, therefore, exactly $V = \frac{4}{3}\pi(\frac{d}{2})^3$, and the surface area is exactly $S = 4\pi(\frac{d}{2})^2$. So, we claim that the SVR (Surface to Volume Ratio) is exactly $\frac{4\pi(\frac{d}{2})^2}{\frac{4}{3}\pi(\frac{d}{2})^3}$, which reduces, after canceling the 4π in the numerator and denominator to $\frac{d^2}{4} \div \frac{1}{3}\frac{d^3}{8} = \frac{d^2}{4} \times \frac{24}{d^3} = \frac{6}{d}$ in unit less numbers.

One big difference between Practical measurements and Ideal measurements are the units used in the measurements.

In ideal measurements we generally do not specify units, but in practical measurements we do. So, as seen above, a sphere with diameter side 1 has a surface to volume ratio (SVR) of $\frac{6}{1}$, or just 6.

But in practical measurements, if the diameter of the sphere is cm, then the ratio is $\frac{6}{1\,cm}$ or $6\,cm^{-1}$. Whilst a much larger sphere, say with a diameter of 1 meter, has a surface to volume ratio of $\frac{6}{1\,meter}$, which we can write as $6\;m^{-1}$ If we want to compare these ratios, then we must put them into the same units. So the bigger sphere has surface to volume ratio of $0,06\;cm^{-1}$ because $\frac{1}{meter} = \frac{1}{100\,cm} = \frac{0{,}01}{cm}$.

On the other hand a smaller sphere, say of a diameter of 1 mm has a surface to volume ratio of $\frac{6}{1\,mm}$ which we must convert to cm for comparison purposes, getting $\frac{6}{1\,mm} = \frac{6}{0{,}1\,cm} = 60\;cm^{-1}$
The surface to volume ratio is increasing as the diameter of the sphere decreases. This fact is missed when we use unit less measurements. Look at the following table:

	Ideal Measurements	Practical Measurements		
Diameter	1	1 $meter$	1 cm	1 mm
SVR	6	0,06 cm^{-1}	6 cm^{-1}	60 cm^{-1}

Table 6.1 Comparing SVR in cm^{-1}

Exercises

6.1 Show that not only the sphere with diameter 1, but also the circular cylinder with height and diameter of 1 has a SVR of 6/1, as well as a cube whose sides are of length 1.

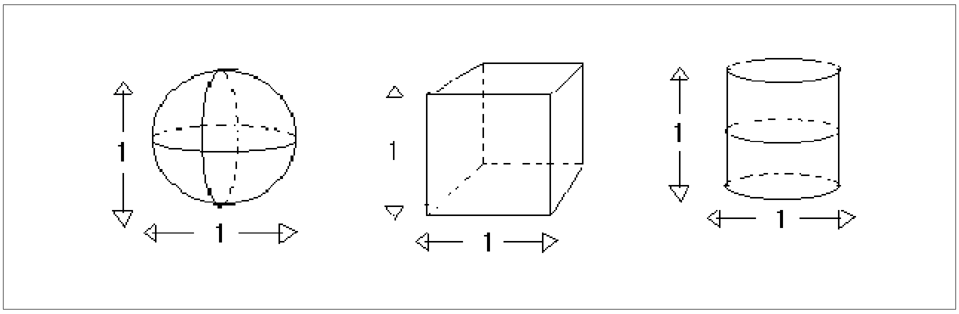


Figure 6.1 Sphere Cube & Cylinder, all with SVR = 6

HINTS: Recall that :
For a sphere of radius r, the surface is $4\pi\,r^2$, and the volume is $\frac{4}{3}\pi\,r^3$.
For a cube of side d, the surface is 6 d^2 and the volume is d^3
For a circular cylinder of radius r and height h, the surface is $2\pi\,r^2 + 2\pi\,rh$, and the volume is $\pi\,r^2h$

Dimensions 1, 2, 3, 4 and beyond

1-D When we measure a *length* of string, a straight line, the *distance* along any straight or curved line or any path or even the perimeter of a plane figure, we are measuring a one-dimensional object (1-D). The units for these measurements are *linear units* (using the power one). In SUI these are Meters (m) and the various multiples such as Kilometers (km) or decimal fractions such as Centimeters (cm), Millimeters (mm), Micrometers (µm), etc. If we include time as a one dimensional quantity, then the units are Seconds, (s) or multiples (or fractions).

2-D When we measure the area of any plane figure, any flat or curved surface of a solid, we are measuring a two-dimensional object (2-D). The units for these measurements are *square units* (using the power two). Thus, we use m^2, km^2, cm^2, ... the squares of the linear units.

3-D When we measure the volume of any solid, or any gas or liquid filling some container, or the space around us or even outer space, then we are measuring three-dimensional objects (3-D). The units for these measurements are *cubic units* (using the power three). Thus, we use m^3, km^3, cm^3, ... the cubes of the linear units.

4-D When we are measuring the location in time of particle in physics or the status of a chemical reaction or the state of a cell in morphogenesis, or any three dimensional object existing during some interval of time, we are measuring a four-dimensional object (4-D).

In the sciences the fourth dimension is usually thought of as the time variable, so the units are not the fourth power of a linear unit, but rather the cube of linear units, used along with the first power of time. In the sciences and in mathematics we represent fourth dimensional quantities as vectors with four components (because we require four measurements to determine the status of the vector). In physics, the vector is the location of the particle, in biology it is the state of the embryonic cell, etc.

In unit less mathematical measurements, the fourth dimension is not considered to be anything special such as time, but it is just another variable and we have no problem using fourth powers, fifth powers, etc. So the dimensions can easily be 4-D, 5-D, ,... n-D for any integer n. By the way, if $n = 0$, then we have zero-dimensional space (0-D), which is just a single point. Even in physics, vectors of any finite dimension can be used to describe various physical states. Sometimes an object is considered to have infinitely many dimensions. One may quote the following definition of physical reality. "Reality is a vector in Hilbert Space." Hilbert Space is an infinite dimensional vector space that has a certain properties involving the measurements of distances.

Dimensions $\frac{1}{2}$, $\frac{3}{4}$, $\frac{x}{y}$, $1\frac{2}{3}$, $1\frac{x}{y}$, $2\frac{x}{y}$, and beyond.

In chemistry, if you try to measure chemical processes that have turbulent reactions, you will find that you may be trying to measure something that has more than 0-D and

less than 1-D. If you try to measure the Indian Ocean coastline of Mozambique with a meter stick you will get an under approximation, because you have left out all the coves and nooks and crannies that don't lie along a straight one meter line. If you try using a 30 cm ruler you can increase your accuracy, but you will still leave out all the nooks and crannies that don't lie along a straight 30 cm line. You will find that the coast line has a dimension of something between 1-D and 2-D. In physics, the study of chaos leads to measurements in fractional dimensions. In meteorology, clouds and certain weather patterns depict conditions whose measurements are fractional dimensions. Trees, mountains, broccoli and cauliflower are all examples of objects that have approximate measurements that are not just 1-D or 2-D or 3-D, but some dimensions in between these whole number dimensions.

The difficulty in trying to measure these strange objects is that you cannot magnify them enough to find any straight parts to measure with a straight ruler. Every magnification of these objects has the exact same chaotic pattern as the original full sized object. Look at broccoli, if you break off a small piece, it looks exactly like the original whole broccoli and if you break off a small piece of the small piece, it looks just like the original small piece. The pattern does not change, no matter what level you are looking at. This property is called *scale-invariant.* Since the pattern doesn't change, then there is no ruler fine enough to get the exact measurement. Roughly speaking you could say it is a surface trying to fill up space. So its dimension is somewhere between 2-D (the dimension of a surface) and 3-D (the dimension of a volume). It has fractional dimension.

Objects that are scale-invariant and have fractional dimension are called **fractals**. See "How to Make a Fractal Snowflake" in Appendix 1.

Putting aside the fractals, let us look at the area and volume of "genuine" 2-D and 3-D objects that maths and science have in common.

Area

Let $\mathcal{F}$ be any <u>2-D object,</u> whether it be flat like a floor or a curved surface like the bonnet of a car. We can also let it have any shape, from a rectangular to a daisy (as long as it is not a fractal). We can get an approximate measure area of $\mathcal{F}$ by overlaying a grid of small squares and counting the number of squares that lie completely within $\mathcal{F}$.

See Figure 6.2 below.

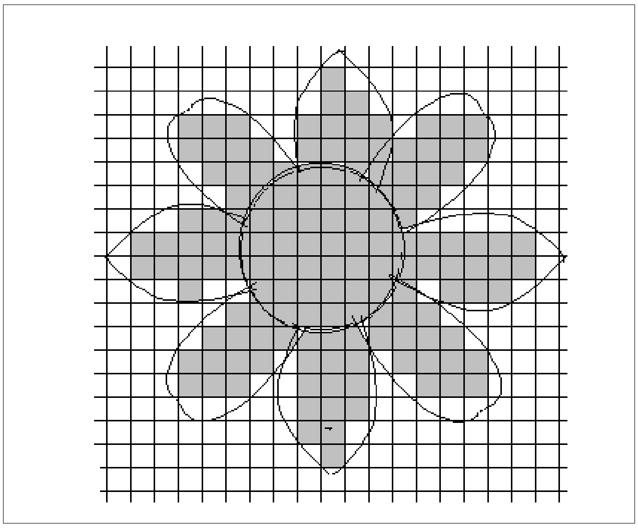


Figure 6.2 Under estimate of the area by the counting the 148 interior squares

Each small square in Figure 6.2 has an area of $\frac{1}{9}\,cm^2$, and there are 148 interior squares. The sum of the interior squares gives us that the under-approximation of the area of the daisy is 148 $\times\ \frac{1}{9}\,cm^2\ = 16.2\,cm^2$.
Now if we add the squares that are partly inside and partly outside of the daisy, we will finish filling up the daisy and then some. Counting these yields 68 more squares. This gives us a total of 216 squares either inside or partially inside of the daisy, as can be seen in Figure 6.3.

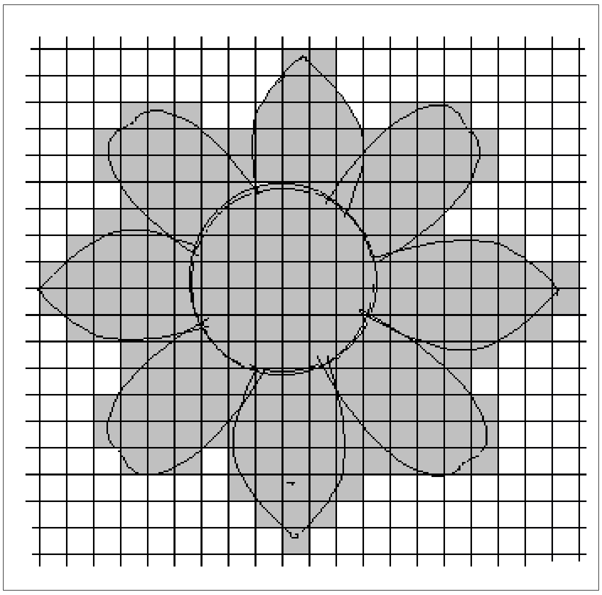


Figure 6.3 Over estimate of the area by the counting the 216 squares totally or partially interior

At $\frac{1}{9}cm^2$area for each square, we get an over approximation of $216 \times \frac{1}{9}cm^2 = 24$ cm^2

The actual area is between these two approximations (16.2 cm^2 and 24 cm^2). So, we take the daisy to be approximately $20\,cm^2$.

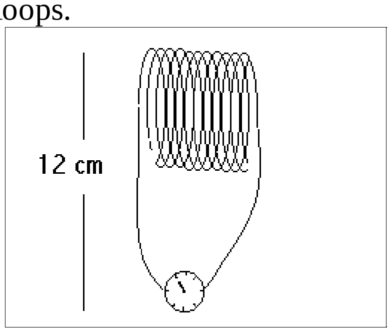
Exercises

6.2 Trace some regular or irregular object from a science or maths class room or from home and use a grid of small squares to get an under estimate of the area, an over estimate of the area, and average these two estimates. If possible, use a formula to obtain the actual area and compare it to your estimates.

6.3 Explain why it is that you can get improved approximations of the area of the daisy by decreasing the size of the small squares in the grid.

6.4 (a) What are the two properties of a fractal?
(b) Why can you not estimate the "area" of a fractal by using a grid of squares?

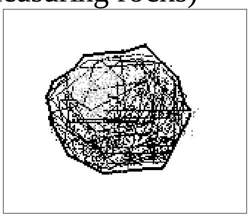
6.5 Use the formulas for area and circumference of a circle on following problems:
(a) A circle of area 300 cm^2 is formed by a rotating object.
Calculate the perimeter.
(b) A piece of wire 100 cm long is coiled to form a circle, calculate the radius.
(c) In a physics experiment, a copper wire of length 3 meters is coiled into 10 circular loops leaving 12 cm at each end to attach to a galvanometer. Calculate the diameter of the loops.



6.6 The water in a bore hole is brought up in a bucket by winding a rope around a cylindrical shaft. If the shaft has a diameter of 11 cm and it takes 52 turns of the crank to bring a bucket up from the bottom, how deep in meters is the bore-hole?

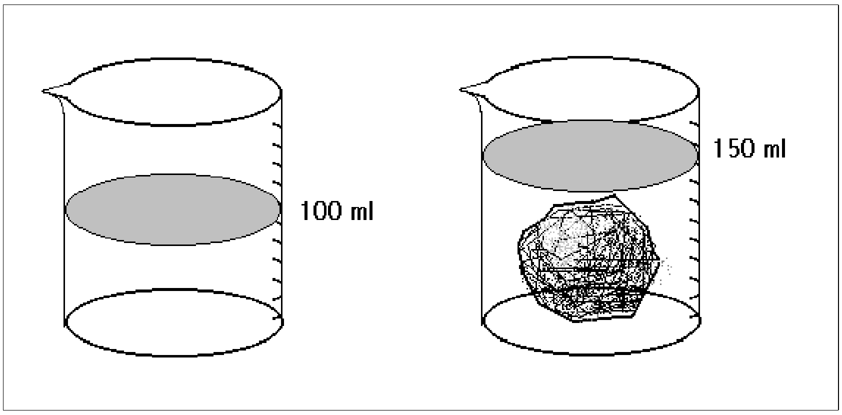
6.7 The rate if production of oxygen gas from a chemical reaction is 5 cm^3/s. The reaction goes on for four minutes. Calculate the volume of oxygen produced.

Mameasurements Matombo (Measuring rocks)

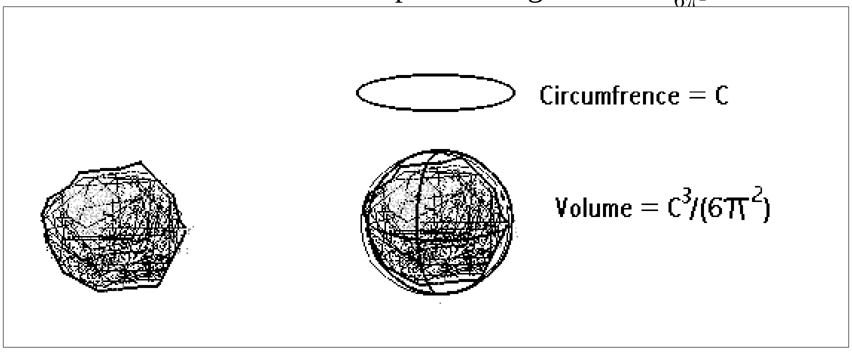


To find the volume of an irregularly shaped solid, such as a rock, we have two ways.

1. We can find the volume by seeing how much water is displaced when the rock is submerged in water.



2. If the rock is approximately spherical in shape we can measure the circumference, C and use the formula for the volume of a sphere and get $Vol = \frac{C^3}{6\pi^2}$.



Exercises

6.8 (a) Given a sphere of circumference C, and given that the volume of a sphere in terms of its radius r is $Vol = \frac{4}{3}\pi r^3$, derive the formula Volume = $\frac{C^3}{6\pi^2}$.

(b) If a rock in the figures has a circumference of 14.4 cm, find its approximate volume, by assuming it is approximately a sphere.

CHAPTER 7 RATIOS

Gradients and slopes.

A ***gradient*** is a number that represents a ***gradual*** change in some variable quantity with respect to another variable quantity. If you are driving up a mountain road, for example, you gradually increase your height above sea-level for every kilometer you travel horizontally. The ratio of change in height divided by the horizontal distance traveled is called the gradient or the ***slope*** of the road.

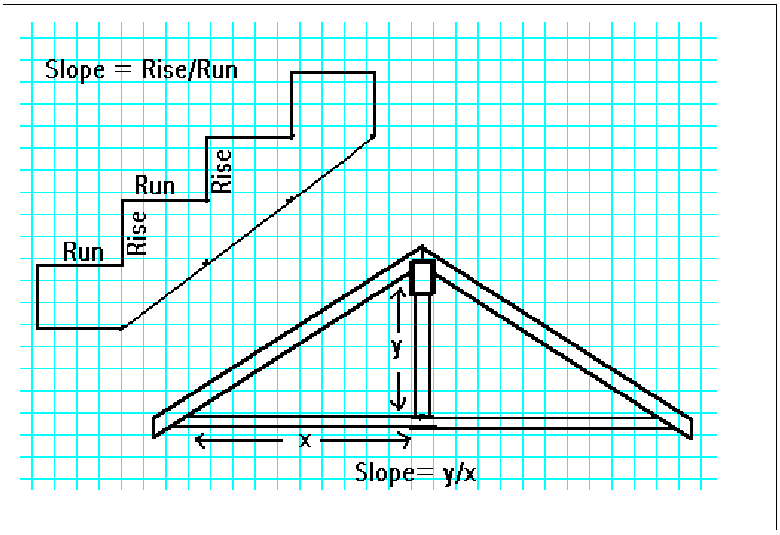


Figure 7.1 Slope of a stair and a roof

Slopes and gradients are the same thing; they are measures of "steepness". Usually we use the word slope in geometric problems and we use the word gradient in science and other types of applied problems. The slope of a roof of a house, for example, is a number that tells you how steep the roof is. In a designing stair case, architects call the slope the "rise-to-run ratio" (the height of each step divided by the horizontal part of each step). See Figure 7.1 above.

To be more precise, we define slope as follows: The slope of a line passing through any two points A and B, not on the same vertical line, is the number m defined by the ratio:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Where A = $(x_1; y_1)$ and B $= (x_2; y_2)$. See Figure 7.2 below.

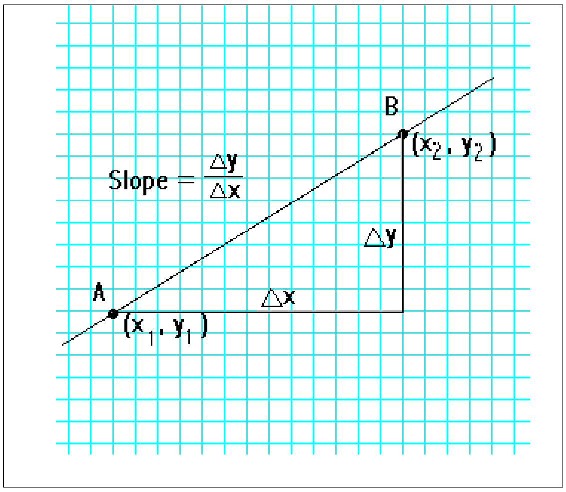


Figure 7.2 Slope of line AB
$\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$

Example: If A is the point with coordinates $(2; 7)$ and B is the point with coordinates $(15; 16)$, then the slope of the line AB is $\frac{16-7}{15-2}$ or $\frac{9}{13} \approx 0,7.$

The reason that we don't allow the two points A and B to be on the same vertical line is that any two points on the same vertical line have the same x-coordinates, $x_2 = x_1$, so $x_2 - x_1 = 0$. Hence m would not be defined.

In science classes and maths classes, we use the notation Δx and Δy to stand for the differences $(x_2 - x_1)$ and $(y_2 - y_1)$, respectively. Thus we may say that the slope of the line AB is defined by $m = \frac{\Delta y}{\Delta x}$. By the way, this same "delta" notation is also useful to express the formula for the distance between the two points A and B, which is

$$Distance(AB) = \sqrt{(\Delta y)^2 + (\Delta x)^2}.$$

(Based upon the Pythagorean Theorem applied to the triangle in Figure 7.2)

Exercises

7.1 (a) Find the slope of the line containing the two points A and B, where A = (0;1) and B = $(\frac{1}{4}\pi; 0.707)$

(b) Find the slope of the line containing the two points B and C, where B = $(\frac{1}{4}\pi; 0.707)$ and C = $(\frac{1}{2}\pi; 0)$

(c) Find the slope of the line containing the two points C and D, where C = $(\frac{1}{2}\pi; 0)$ and D = $(\frac{3}{4}\pi; -0.707)$.

7.2 (a) If A and B are two points of the graph of $y = cos(x)$ and the x-$coordinate$ of A is $x_1 = 0$, and the x-$coordinate$ of B is $x_2 = \frac{1}{4}\pi$, find the slope of AB.

7.3 If P and Q are two points on the same horizontal line, with $P = (p, y)$ and $Q = (q, y)$, find the slope of the line PQ.

Pressure Gradient

Here is an example of a constant gradient in science. The definition of pressure states that if a an object of mass W is resting on a region of area A, then the pressure P, on the region is given by the equation: $P = \frac{kW}{A}$, where k is a constant. Thus if we hold the area at a constant and increase the mass then the pressure will increase as a multiple of W.

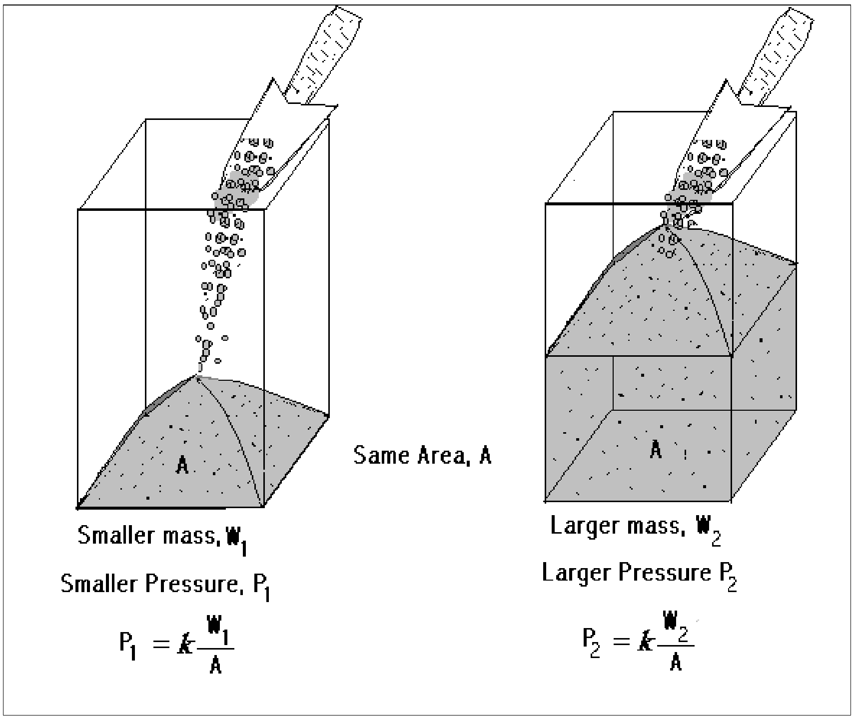


Figure 7.3 Pressure over a fixed area, P = $k\frac{W}{A}$

In Figure 7.3 the pressure is a linear function of the mass, and the gradient $\frac{\Delta P}{\Delta W}$ is the constant $\frac{k}{A}$. This is derived as follows, The change in pressure is:

$$\Delta P = P_2 - P_1 = \frac{kW_2}{A} - \frac{kW_1}{A} = \frac{k}{A}(W_2 - W_1)$$

whilst the change in mass is

$$\Delta W = W_2 - W_1$$

Therefore,

$$\frac{\Delta P}{\Delta W} = \frac{\frac{k}{A}(W_2 - W_1)}{(W_2 - W_1)} = \frac{k}{A}.$$

The following graph depicts the pressure at as we gradually add mass (sand), the slope of the line is the constant pressure gradient.

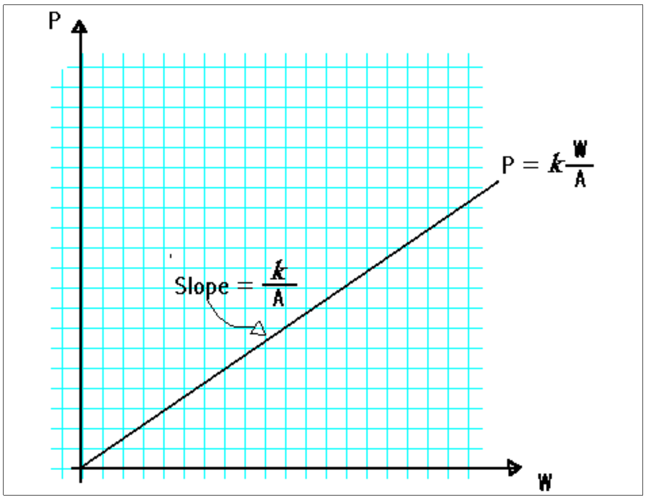


Figure 7.4 Pressure, P as a function of mass, W

Notice that the pressure itself is not a constant, it changes with the change in mass, but its gradient *is* a constant.

Boyles' Law for gases, the volume gradient.

Pressure affects the volume of a fixed amount of gas in a non-linear way. $V = \frac{k}{P}$, and the gradient is not constant.

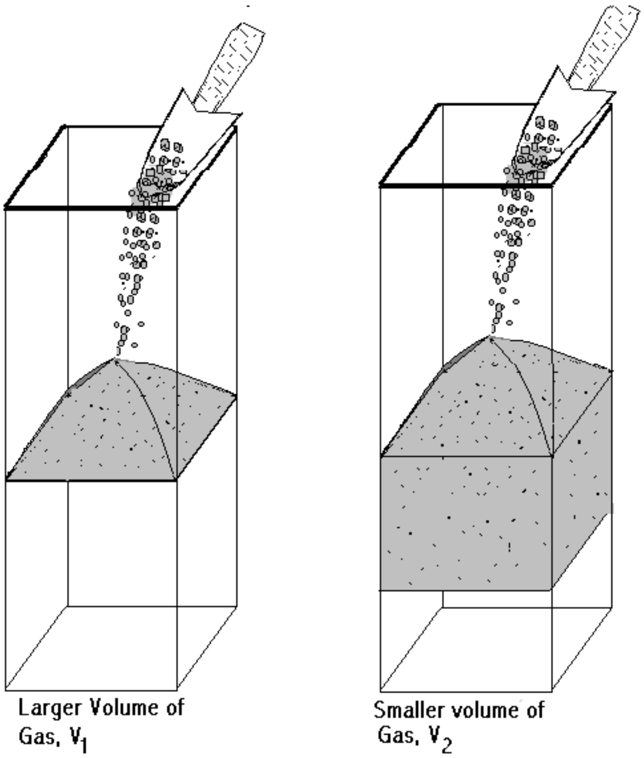


Figure 7.5 Volume decreases as pressure increases

In Figure 7.5, we see that the larger pressure makes the gas have a smaller volume. If we use the equation $V = \frac{k}{P}$, then the two volumes are $V_1 = \frac{k}{P_1}$ and $V_2 = \frac{k}{P_2}$.Now if we calculate the gradient, then we will get $\frac{\Delta V}{\Delta P} = \frac{-k}{P_1 P_2}$, which is not a constant, but depends upon the two pressures, P_1 and P_2. The graph of volume vs. pressure depicting the change in volume as we increase the pressure (by gradually adding sand) is as follows:

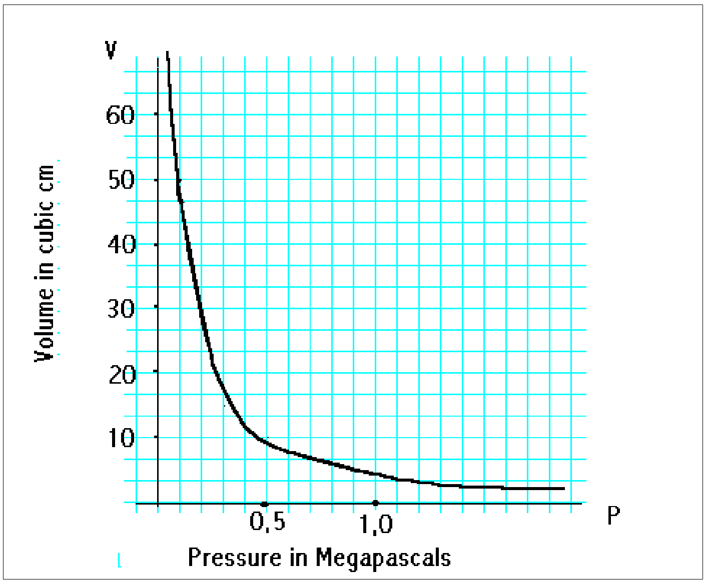


Figure 7.6 Volume, V as a function of pressure, P

The graph is not a straight line and does not have a constant slope, so the gradient of volume with respect to pressure is not constant.

Exercises

7.4 Using the formula for Volume as a function of Pressure, $V = \frac{k}{P}$ to show that the gradient, $\frac{\Delta V}{\Delta P}$ of V as P changes from P_1 to P_2 is $\frac{-k}{P_1 P_2}$.

Change of coordinates.

In the sciences, we sometimes change the variables in order to simplify a graph. For example, if we take the graph in Figure 7.6 and let $y = V$ and we let $x = \frac{1}{P}$, (x is 1 divided by the pressure, then on the x-$axis$ x will be 2 when $P = 0.5$, x will be 9, when $P = .111$...and the equation $V = \frac{k}{P}$ will become $y = kx$. We plot Boyle's law in this transformed coordinate system as follows.

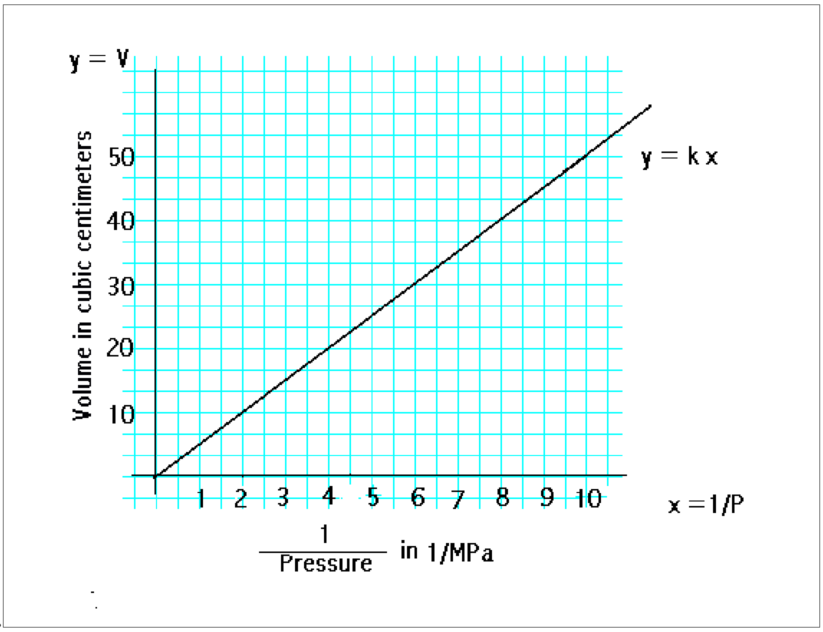


Figure 7.7 The Volume to pressure graph rendered by changing the x variable to $1/P$.

In this straight line graph the gradient between two points (x_1, y_1) and (x_2, y_2) is k. That is, $\frac{\Delta y}{\Delta x} = k$. But does that mean that the volume increases at the rate k times the increasing pressure? ***No!*** It means that the volume increases at a rate of k times the *reciprocal of the pressure* $(\frac{1}{P})$. In other words, $\frac{\Delta y}{\Delta x} = k$, but $\frac{\Delta y}{\Delta \mathrm{P}}$ is still $\frac{-k}{P_1 P_2}$. This is because when we set $x = \frac{1}{P}$ we are also saying $P = \frac{1}{x}$, so

$$\Delta P = P_2 - P_1 = \frac{1}{x_2} - \frac{1}{x_1}$$

Combining $\Delta y,\ \Delta x, \Delta P$, we get $\frac{\Delta y}{\Delta \mathrm{P}} = \frac{\Delta y}{\Delta x} \times \frac{\Delta x}{\Delta P} = k \times \frac{\Delta x}{\Delta P} = k\frac{x_2 - x_1}{\frac{1}{x_2} - \frac{1}{x_1}}$.

If you complete this calculation you will get the afore-mentioned formula for $\frac{\Delta y}{\Delta \mathrm{P}}$. Both Figure 7.6 and Figure 7.7 give you exactly the same information. The advantage of Figure 7.7 is that it is easier to draw a straight line. But on the other hand, the disadvantage is that it is more difficult to get the scale on the x-$axis$ correct.

Exercises

7.5 Complete the calculation showing the $\frac{\Delta y}{\Delta \mathrm{P}} = \frac{-k}{P_1 P_2}$, starting with $\frac{\Delta \mathrm{y}}{\Delta P} = k\frac{x_2 - x_1}{\frac{1}{x_2} - \frac{1}{x_1}}$

7.6 If on some planet or asteroid, the distance D, fallen in time t, is given by the equation $D = kt^2 + 5$

(a) For two points $(t_1,\ D_1)$ and (t_2, D_2), Find ΔD and Δt and the gradient $\frac{\Delta D}{\Delta t}$, simplify.

(b) Change the variables to let $y = D$ and $x = t^2$, then for $x > 0$, $t = \sqrt{x}$. Now the equation is $y = kx + 5$. Show that $\frac{\Delta y}{\Delta x} = k$, but $\frac{\Delta y}{\Delta t} = k(t_2 + t_1)$

A straight line has a constant slope, a curved line does not have a constant slope. What do we mean by the slope of a curved line anyway? We cannot talk about the slope of the whole curve at once, but instead we talk about the slope of the curve ***one-point-at-a-time***. When we refer to the slope of a curve, we are referring to "instantaneous slope at a given point of the curve." We are measuring how steep the curved graph is at a point, and we do this by finding the slope of the line that is tangent to the graph at that point. See Figure 7.3 below.

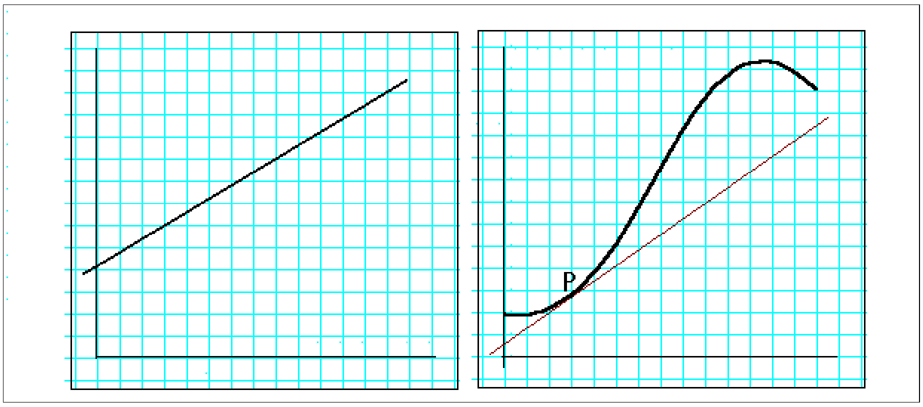


Figure 7.8
The slope of the straight line is constant throughout the whole graph, but the slope of the curve at P is the slope of the line touching the curve at that point.

Exercises

7.9 If a stair case has ten steps and the height of each step is 17 cm and the horizontal part of each step is 21 cm,

(a) What is the rise to the run?
(b) What is the vertical height from the floor to the top of the stair case?
(c) What is the horizontal distance from one end to the other end of the stair case?

7.10 In Figure 7.8 find the slope of the line tangent, at P, for the graph on the right. You can do this by starting at P and moving horizontally to the right counting five squares (this will be the "run"), then move vertically upward counting squares until you hit the tangent line. What ever you get on this upward move will be the "rise".

A note on the general interpretation of "slope".

In maths the ratio of y divided by x is a pure number $\frac{y}{x}$. Thus, in Figure 7.9(a), the rectangle ABCD, with height y and base x, the pure number ratio $\frac{y}{x}$ is the slope of the diagonal (the line AC).

But if we assign some units to these variables, then their ratio may have some meaning and is subject to analysis and interpretation. For example if the base, x represents time in seconds and the height, y represents distance in meters, then the slope, $\frac{y}{x}$ of the diagonal, AC is the speed, $\frac{distance}{time}$ with units in m/s. See Figure 7.9(b).

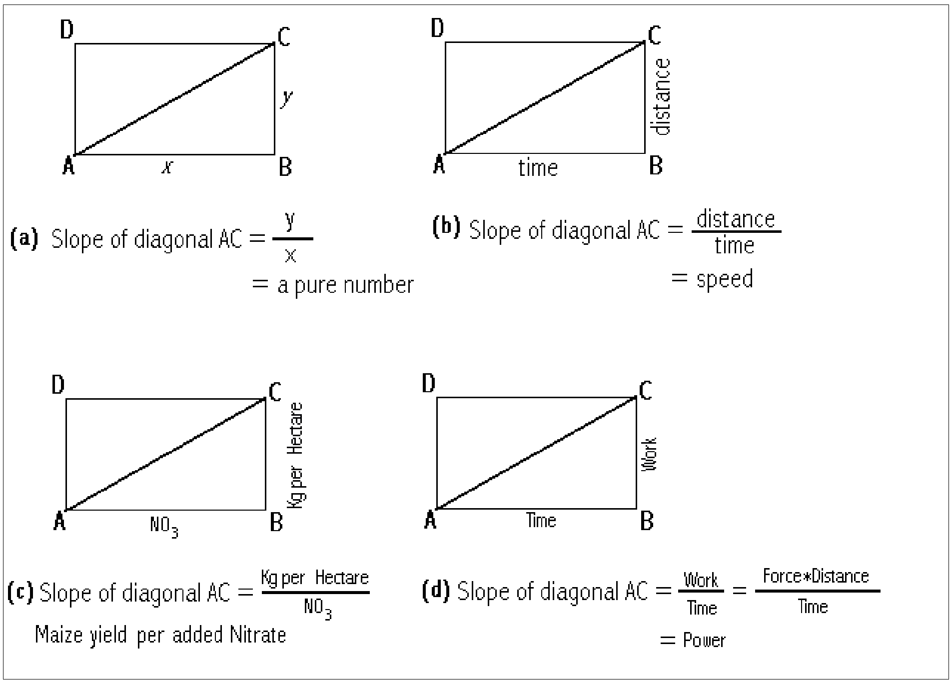


Figure 7.9 Various interpretations of slope determined by the units of the base and the height of the rectangle

In Figure 7.9(c) the slope of the diagonal is "Kg of maize per hectare per added Nitrate" because $x =$ amount of NO_3, and $y =$ Kg of maize per hectare. In Figure 7.9 (d), we have that the slope of the diagonal is power, since the base,x is in time units and the height y, is in work and Work/Time $=$ Power.
We may draw other rectangles, in which the slope of the diagonal is "time of chemical reaction per grammes of a catalyst", or " sale of mangos per amount of yellow color". We simply take the ratio of the units on the vertical and horizontal sides of the rectangle.

Exercises
7.11 What units are assigned to the slope of the diagonal when the base x is time in seconds and the height y is in velocity of a particle in meters/sec?
7.12 What units are assigned to the slope when $x =$ speed in Km/hour, and $y =$ distance in Km?

Tichaonana. In this chapter, we have looked at gradients (the rate of change of one variable with respect to another). This is one of the basic concepts studied in calculus. In the next chapter we will look at products and sums of products. This is a calculus concept that lets you study Work, Power, Total populations, and other applications that involve finding sums of small pieces.

CHAPTER 8
PRODUCTS

Area is the product of two lengths, $A = l \times w$.
Work is the product of Force and Distance, $W = F \times D$.
Momentum is the product of mass and velocity, $\mu = m \times v$.

These simple formulas are used to solve problems when the two factors being multiplied together are both assumed to be constant, throughout the problem.

Example

Find the work done when a force of 50 newton moves a box a distance of 8 meters..
The answer is 50 newtons $\times$ 8 meters, or 400 Nm = 400 Joules.

A newton (1N) is a unit of force which is, itself, a product.
$Force = mass \times acceleration.$ In free space a newton is the force required to propel a mass of one kg with an acceleration of one meter per second per second.
$N = 1kg \times 1\,meter/sec^2$. On earth, a mass of 100 gm pulled down by the gravitational acceleration of $10\,meters/sec^2$ is a force (a weight) of one newton because $100\,gms \times 10\,m/se\ c^2$ = the same as a unit of force in free space.

A problem such as that in the above example is a simplified version of a more realistic problem in which the force is not constant over every meter. When a rocket is launched the force needed for lift off is very large and the work done in raising the rocket the first few meters is larger than the work done over the same distance in the upper atmosphere because the force being applied up there is much less. So the total work done in lifting the rocket is the sum of the varying amounts of work done throughout the voyage (large forces times small distances at first, then small forces times large distances later.)

Example

Find the work done in the following problem. A box is pushed 8 meters along a downward sloping floor. The initial force is 50 newtons, but the force is reduced by 5 newtons as the box travels through any given meter. Thus, through the first meter the box is pushed by a force of 50 newtons, during the second meter the box is pushed with force of 45 newtons, etc.

Solution:

The force applied depends upon where the box is (how many meters it is from the staring point). We will use the notation F_x to stand for the force applied over the xth meter. Thus, applied over the first meter, denoted by F_1, is 50N. The force, F_2 applied over the second meter is 45N, then over the third meter the force, F_3, is 40N, and so on. The general formula would be found by reducing the initial 50N by 5N each time we pass over a meter. That is $F_x = 50\ - 5 \times (x - 1)$. The work done over each meter is obtained by multiplying the force for that meter by the distance of 1 meter as follows:

Work done in the first meter $= F_1 \times 1$
Work done in the second meter $= F_2 \times 1$
Work done in the third meter $= F_3 \times 1$
etc., until finally,
Work done in the eight meter $= F_8 \times 1$

The total work, W done is :

$$W = F_1 \times 1m + F_2 \times 1m + F_3 \times 1m + ... + F_8 \times 1m$$

which we write in summation notation as:

$$W = \sum_{x=1}^{8} F_x \times 1m$$

Where the capital Greek letter sigma $\sum$ means "sum". The summand $(F_x \times 1m)$ tells you **what** to sum. The lower index $x = 1$ tells you to **start** the sum with $x = 1$ and the upper index 8 tells you to **stop** the sum when $x = 8$.
Here we get:

$$W = \sum_{x=1}^{8} F_x \times 1m = 50Nm + 45Nm + ...15Nm = 260Nm = 260\,joules.$$

Exercises
8.1 In Graph A of Figure 8.1, the horizontal line shows a constant force of 50 newtons applied over a distance of 8 meters. In Graph B the "stair-step" line (sometimes called a step-function) represents a declining force of $50 - 5(x-1)$ newtons applied over each of x meters where x runs from 1 to 8
(a) Find the area under each of these lines.
(b) What does the area represent?
(c) What are the units of the area?

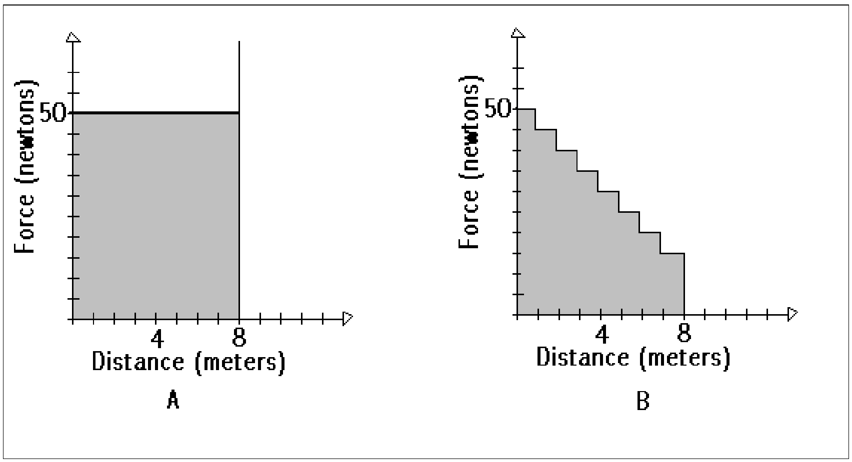


Figure 8.1

8.2 A piano weighing 790Kg is raised from the sidewalk to a second floor window, 13 meters above the sidewalk. How much work is done?

8.3 A fishnet has captured 140Kg of fish, but just as the net comes out of the water it also contains 80kg of water (so the total weight of the fishnet and its contents is

220Kg). The boat deck is 10 meters above the water level and as the net is raised it loses10kg of water per meter raised. How much work is done by the hoist in bringing up the fish (and water until it has all run out)?

8.4 A coordinate system is set up so that it has, on its x axis, units of time and on its y axis, units of velocity. A graph is drawn that represents a particle's velocity at any time (See Figure 8.2)

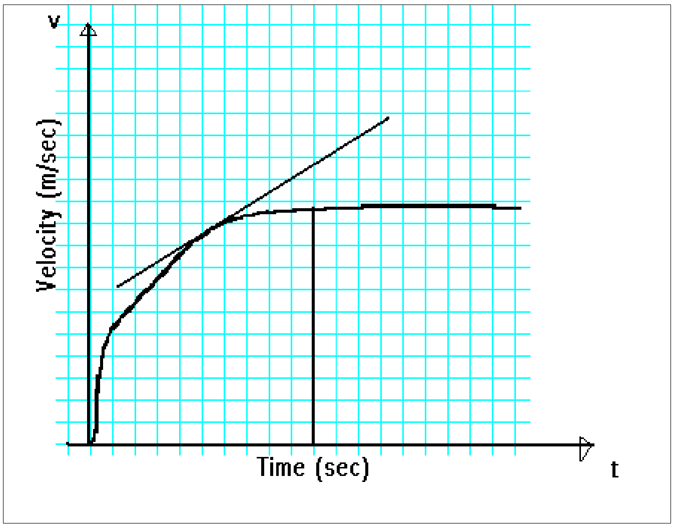


Figure 8.2

(a) What are the units of the area under the curve?
(b) What are the units of the slope of a line tangent to the curve at some point, P.

A Note on the general interpretation of "Area"
Strange as it may seem, we can think of ***distance***, or work, or momentum, etc. as area! For example, distance is the product of two things, speed and time, $d = s \times t$. As for work, momentum, etc. we can say that these quantities represent ***an interpretation*** of the geometric concept of ***area***. Physically, we don't usually think of work as an area, but if we draw a graph in which we plot Force on one axis and Distance on another, then the area under the curve represents Work because this area is obtained by multiplying the units on the two axes.

Question: How can area be interpreted as an electrical charge? Answer: consider a rectangle whose base is in units of time and whose height is in units of current (charge per unit time). To find the **area** we multiply the base times the height, and so when we multiply the units, $time \times \frac{charge}{time}$, the "$times$" cancel, leaving **charge**.

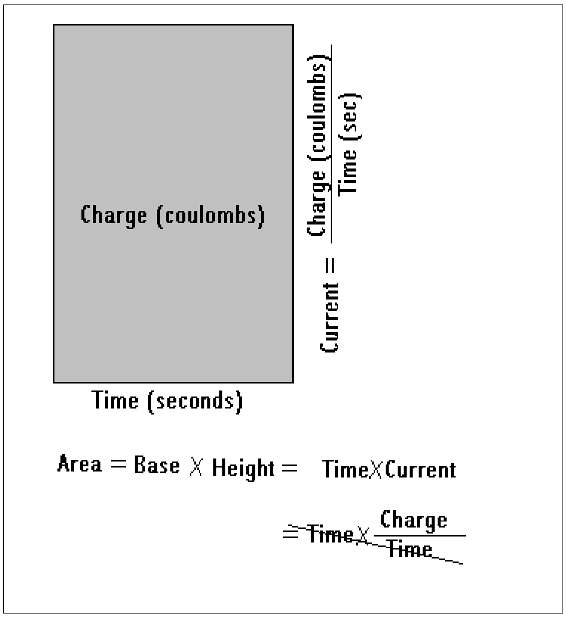


Figure 8.3 How area can be an electrical charge

Exercises

8.5 What interpretation does area have if the base of a rectangle is in years and the height of the rectangle is annual rainfall in cm/year.

8.6 Refer to Figure 8.4 below and find the volume of water in the cylinder of radius 7 cm as a product of depth d and the cross-sectional area A = $\pi \times 7^2$.

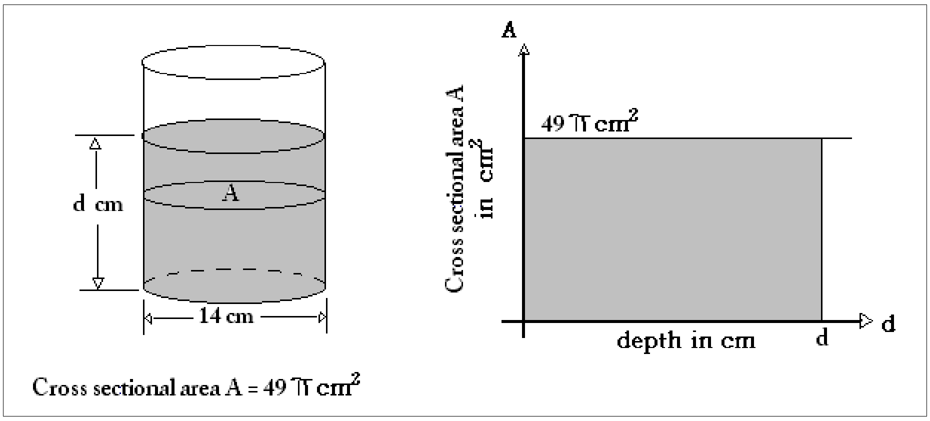


Figure 8.4 Volume of a cylinder as area under a line.

8.7 The aluminum cooling pipe shown in Figure 8.5 below, has dimensions as indicated. It has a constant cross section (or end face) consisting of a square with semi-circles removed from the middle of each side and a circular hole. All the circles have radius 2 cm and each side of the square is 12 cm. The pipe is 30 cm long.

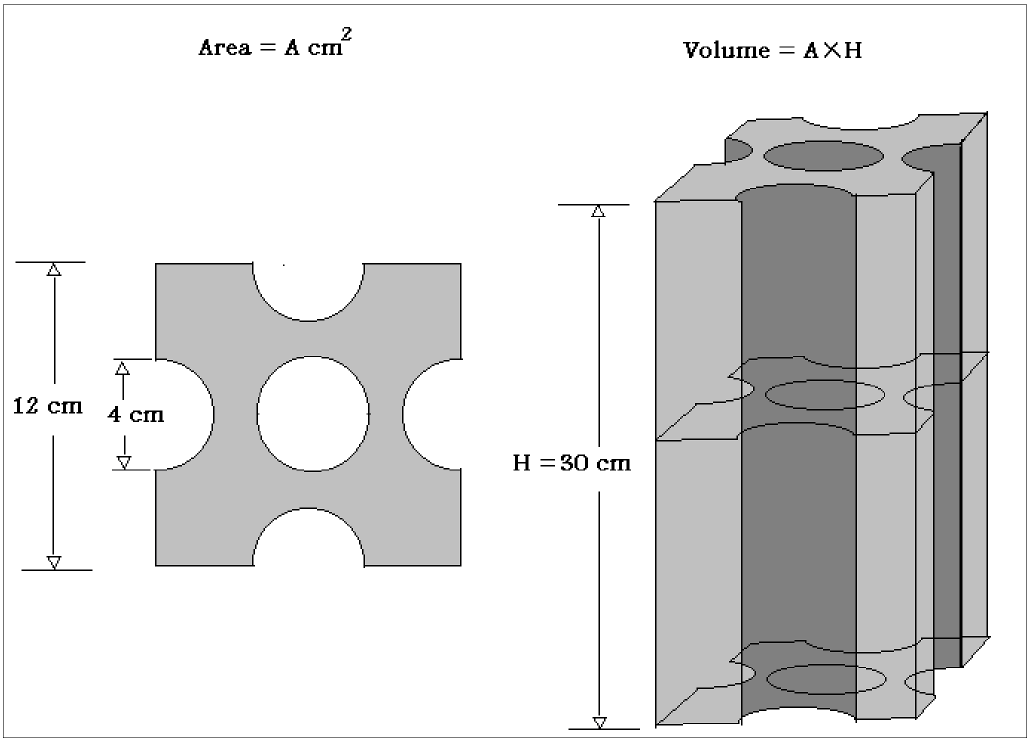


Figure 8.5 An aluminum cooling pipe with a constant cross sectional area of A cm^2 and length H cm, the volume V is A × H.

(a) Find the area, A of the cross section. (End face)
(b) Find the volume of Aluminum needed to make this section of pipe.
(c) Find the outside surface area of the Aluminum.

A note on what information you can get from a graph.
There are three pieces of information you can get from a graph in which the x axis and y axis have been given units. If a graph has units u_1 on the x axis and units u_2 on the y axis, and y is related to x by the equation $y = f(x)$, then you can find the following three pieces of information:

The Present:

The instantaneous value of y at the current value of x. Because you can find $u_2 = f(u_1)$.

The Future:

How y is changing with respect x at this very instant. That is, the direction that the graph is going. Because you can find $\frac{\Delta u_2}{\Delta u_1}$.

The Past:

The total value that y has had in the past from back when $x =$ some earlier value up until the current value of $x = u_1$. Because you can find the area $u_2 \times u_1$ under the curve from some previous value of x up to the current value, $x = u_1$.

Extracting this kind of data from a graph is called analysis (or calculus); it is a useful tool in both science and in maths.

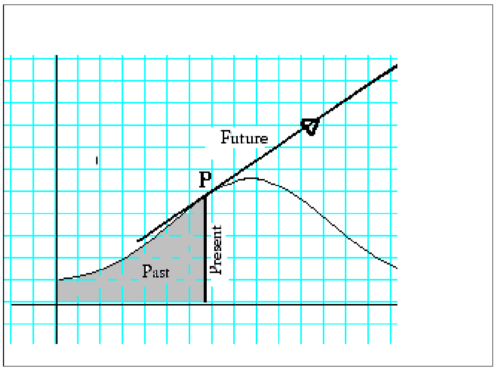


Figure 8.6 Three pieces of information at a given point, P.

ANSWERS TO SELECTED EXERCISES

Chapter 1

1.3(a) Admires's score was better than Matthews because it took fewerstrokes for the five holes.

1.4 (b) Because you cannot conveniently epress faractions in talley notation.

(c) Chipo's score was best because her average score per each hole was smallest.

Chapter 2

2.2 (a)

x	0	1	2	3	4	5
x^2	0	1	4	9	16	25

2.2 (b) The entries in row 2 are the squares of corresponding entries in row 1.

2.3 (a) Increasing the length of the pendulum decreases the number of swings per minute. (e) Time taken by pendulum A to make one swing is $\frac{60}{62}$second. (Approximately one second per swing). (f) Time per swing for pendulum E is approximately 1,54 seconds. (g) A 90 cm pendulum would between 29 and 30 swings per minute.

2.4 (b) Population size at 8:00 Pm is approximately 180 cm^2 (c) At 3:30 it is approximately 29 cm^2

Chapter 3

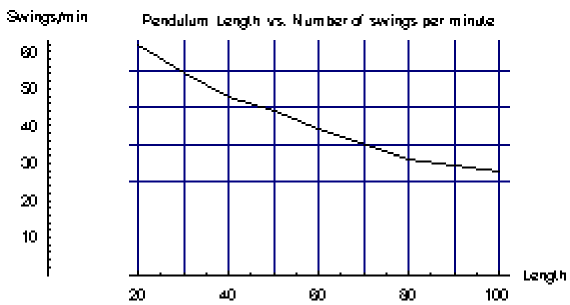
3.6 (a)

x	0	1	2	3	4	5	6	7
y	1	$1,8$	$4,8$	$6,5$	$4,8$	$1,6$	$0,8$	$0,9$

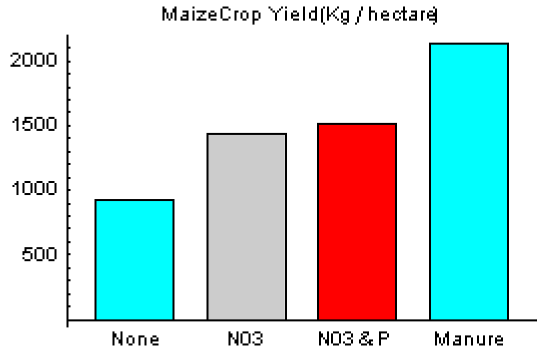
(b)

x	y
0	1
1	$1,8$
2	$4,8$
3	$6,5$
4	$4,8$
5	$1,6$
6	$0,8$
7	$0,9$

3.8



3.10



Chapter 4

4.1 (a) $x + (-x) = 0$; Identity,
(b) $x = 3 - 2x$; (b) conditional,
(c) $y = mx + b$; conditional,
(d) $\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$; contradiction,
(e) $3xy + 7 = 7xy + 3$; conditional,
(f) $x^2y + x^2y = 2x^2y$; Identity,
(g) $x(y - z) = xy - xz$; Identity,
(h) $x + 10 = 2x$; conditional,
(i) $x + 10 = x$; contradiction
(j) $x^2y + x^2y = x^4y$; conditional

4.4 Assuming that there are some numbers satisfying the expression $\frac{1}{x} + \frac{1}{y} = \frac{2}{x+y}$ yields $\frac{x+y}{xy} = \frac{2}{x+y} \Rightarrow (x+y)(x+y) = 2xy \Rightarrow x^2 + y^2 = 0 \Rightarrow x = 0$ and $y = 0$. neither of which may be used in the original equation because of division by zero.

4.7 (c) $V = a_1x + a_2x_2 + a_3x_3$ represents the total value of the three named goods in stock.

4.8 (c) The number of resident adult visitors i 379.

4.13 Surface area $= 0,023mm^2$ or $23\,000\mu^2$

4.14 (a) $y = 2 \times e^{1.3x}$, an exponential growth curve.
(b) $y = 2 + Sin(2x + 1)$, a periodic curve.
(c) $y = 2^{-.5x}$, an exponential decay curve.
(d) $y = (x - 3)^2$, a parabola (polynomial equation of degree 2)
(e) $y = log_5(x)$, a logarithmic function.

Chapter 5

5.1 (a) Eight times heavier, (b) $k = 0,13$ (c) $W(14) = 0,13 \times 2744 = 356\,g$, $W(28) = 2\,853\,gm$

5.2 (a) $778N/m^2$ (b) 778 pascals

5.5 (a) $L(16) = 4$, (b) $L(17) \approx 3,53$

Chapter 6

6.1 For a cylinder, $\frac{surface}{volume} = \frac{2\pi r^2+2\pi rh}{\pi r^2 h}$ and here: $r = \frac{1}{2}$, $h = 1$,
so $\frac{2\pi r^2+2\pi rh}{\pi r^2 h} = \frac{2r+2h}{rh} = \frac{1+2}{\frac{1}{2}} = 6$. For a cube $\frac{surface}{volume} = \frac{6d}{d^2}$ Here $d = 1$.
For a sphere $\frac{surface}{volume} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{6}{r}$, etc.

6.5 (a) $2 \times \sqrt{300\,\pi} \approx 61{,}4\,cm$, (b) $\frac{100\,cm}{2\pi} \approx 15{,}9\,cm$, (c) 24 cm used up in the attachment to the galvanometer, the other 276 cm are coiled into 10 circular loops. Their diameters are approximately $\frac{27{,}6}{\pi} \approx 8{,}79\,cm$

6.6 $11 \times \pi \times 52 \approx 1797\,cm = 17{,}97\,metres$

6.8 (a) Volume of sphere $= \frac{4\pi r^3}{3} = \frac{2\pi r \times 2r^2}{3} = \frac{2\pi r \times 2 \times 2\pi^2 r^2}{3 \times 2\pi^2} = \frac{2\pi r \times 2\pi r \times 2\pi r}{3 \times 2\pi^2} = \frac{c^3}{6\pi^2}$ (Shorai's method). (b) $50{,}4\,cm^3$

Chapter 7

7.1 (a) gradient $= \frac{1-0{,}707}{0-\frac{\pi}{4}} \approx -0{,}373$

7.9 (a) $\frac{17}{21}$ (b) 170 cm (c) 210 cm

7.11 Slope = meters/second^2. Here, slope = acceleration.

7.12 Slope = Km/(Km/hour) = hours. Here, slope = time.

Chapter 8

8.1 (a) Graph A: 400 Nm, Graph B: 260 Nm (b) Work (c) Nm or joules

8.2 $7900\,N \times 13\,m = 102\,700 Nm = 102{,}7\,Kilojoules$

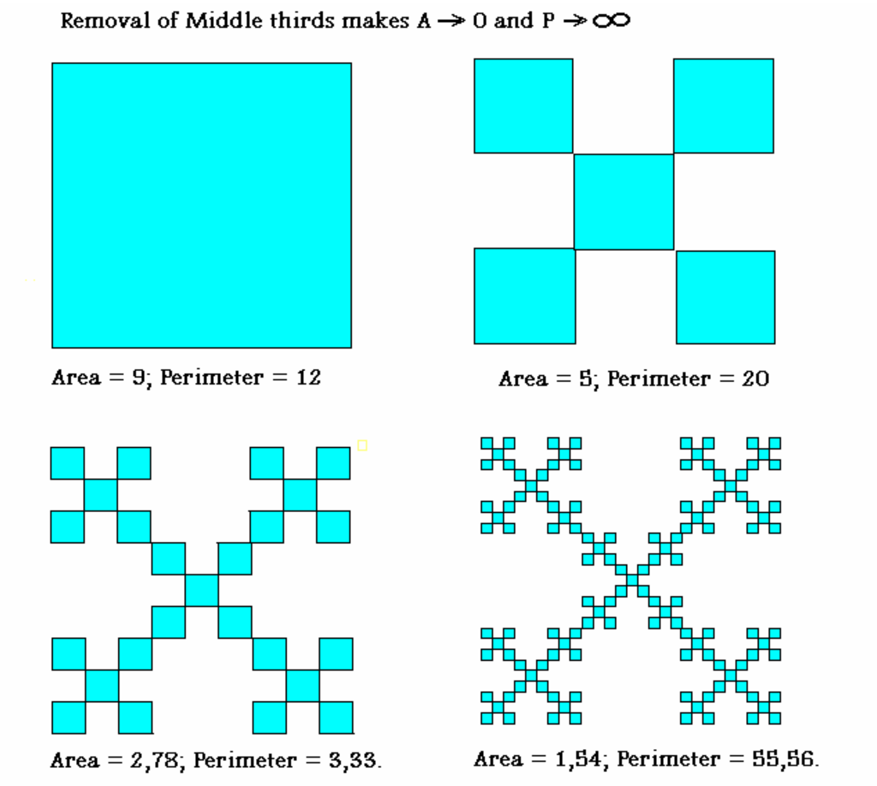
8.3 Work done in the first 8 meters: $\sum_{x=1}^{8}(2200 - 100(x-1)) = 14\,800 Nm$, work done in the last 2 meters (with fish only) is $1400N \times 2m = 2\,800Nm$.
Total work done $= 14\,800\,j + 2\,800 j = 17\,600\,j$

8.7 (a) Cross Sectional Area $= 144cm^2 - 3 \times \pi \times 4cm^2 \approx 106{,}3\,cm^2$
(b) Volume of aluminum $= 30cm \times 106{,}3cm^2 \approx 3189\,cm^3$
(c) Outer perimeter the cross section is $(32 + 8\pi)cm$.
Lateral surface area is 30$cm \times (32 + 8\pi)cm \approx 1714\,cm^2$.
If the areas of the end faces are added, then the total area is
$1714 + 2 \times 106{,}3 = 1926{,}6\,cm^2$.

Appendix

THE MAKING OF A FRACTAL SNOWFLAKE

Step 1. Start with a square of dimensions 3 by 3. The area, A, at this stage is 9, and the perimeter, P, is 12. The linear units may be anything you wish, say cm. Thus, A = 9cm^2, and P = 12 cm.
Step 2. Remove four one-by-one squares around the large square. You are left with five small (one-by-one) squares, thereby reducing A from 9 to (5/9) of 9 and P from 12 to (5/3) of 12. That is the new A is 5cm^2, and the new P is 20cm. We call this "removing the middle third around the perimeter".



Step 3. Remove the middle third around the perimeters of each of the remaining squares. You will reduce the area to (5/9) of the previous A and increase the perimeter to (5/3) of the pervious P. Thus the new A = (5/9) $\times$ 5 $\approx$ 2,78 and the new P $=$ (5/3) $\times$ 20, or approximately 33,33.

Step 4. Remove themiddle third around each of the remaining squares getting $A = (5/9) \times 2,78$, and $P = (5/3) \times 33,33$. Continuing this process forever produces a figure with zero area and infinite perimeter.