Fourier series coefficients

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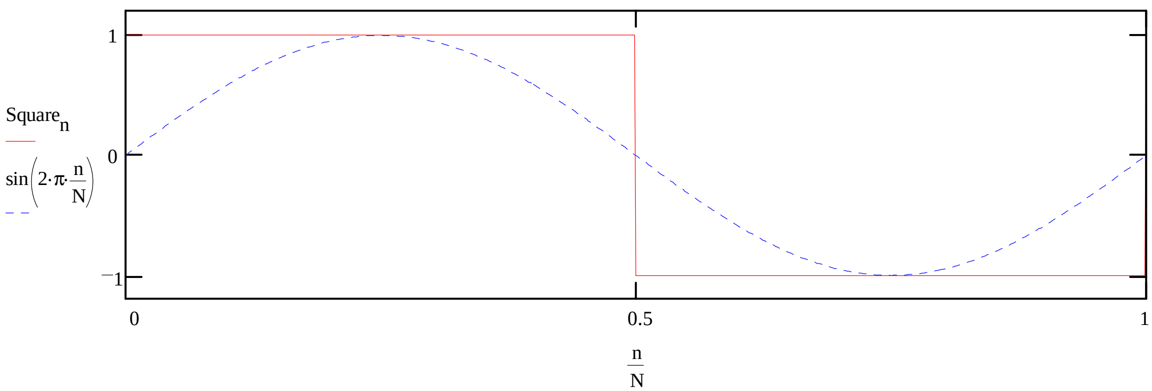
$Square_n := 1 - 2 \cdot floor\left(\frac{n \cdot 2}{N}\right) \quad Square_N := 1$



$\frac{\pi}{2} \cdot \frac{1}{N} \cdot \sum_{n=0}^{N-1} Square_n \cdot \sin\left(2 \cdot \pi \cdot \frac{1}{N} \cdot n\right) = 1$ =1/1

$\frac{\pi}{2} \cdot \frac{1}{N} \cdot \sum_{n=0}^{N-1} Square_n \cdot \sin\left(2 \cdot \pi \cdot \frac{3}{N} \cdot n\right) = 0.333$ =1/3

$\frac{\pi}{2} \cdot \frac{1}{N} \cdot \sum_{n=0}^{N-1} Square_n \cdot \sin\left(2 \cdot \pi \cdot \frac{5}{N} \cdot n\right) = 0.2$ =1/5

$\frac{\pi}{2} \cdot \frac{1}{N} \cdot \sum_{n=0}^{N-1} Square_n \cdot \sin\left(2 \cdot \pi \cdot \frac{7}{N} \cdot n\right) = 0.143$ =1/7

Fourier series

$$C(k) = \int_0^1 F(t) \cdot e^{j \cdot 2 \cdot \pi \cdot k \cdot t} \, dt$$

Discrete Fourier Series

$$C(k) = \frac{1}{N} \cdot \sum_{n=0}^{N-1} F(n) \cdot e^{j \cdot 2 \cdot \pi \cdot k \cdot \frac{n}{N}}$$

where:

$e^{j \cdot x} = \cos(x) + j \cdot \sin(x)$

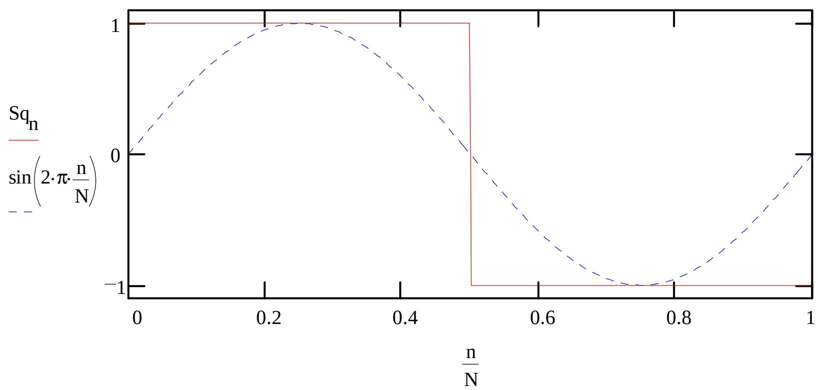
k is the frequency

n is "time"

Do FFT on Square waveform:

$n := 0..N-1 \quad Sq_n := Square_n \quad Sq_0 := 0 \quad Sq_{\frac{N}{2}} := 0$

$Spectrum := FFT(Sq)$



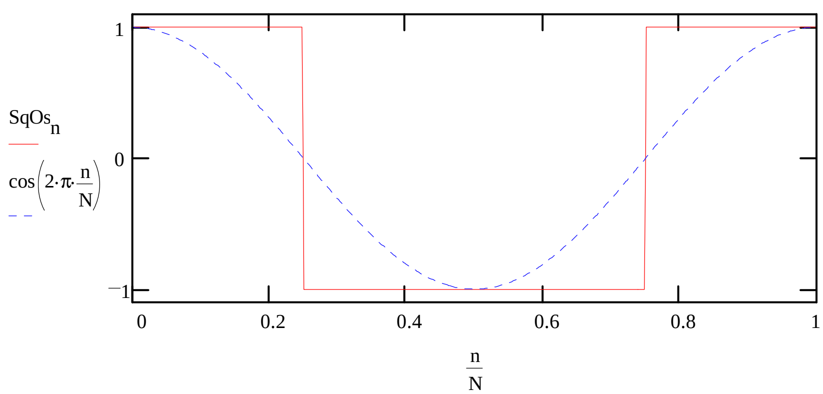
$\frac{-\pi}{2}\cdot Spectrum =$

	0
0	0
1	1i
2	0
3	0.333i
4	0
5	0.2i
6	0
7	0.143i
8	0
9	0.111i
10	0
11	0.091i

Offset in time domain = phase change in frequency domain:

$i := 0..\frac{3}{4}\cdot N-1 \quad SqOs_i := Sq_{i+\frac{N}{4}} \quad i := \frac{3}{4}\cdot N..N-1 \quad SqOs_i := Sq_{i-\frac{3}{4}\cdot N}$

$Spectrum := FFT(SqOs)$



$\frac{\pi}{2}\cdot Spectrum =$

	0
0	0
1	1
2	0
3	-0.333
4	0
5	0.2
6	0
7	-0.143
8	0
9	0.111
10	0
11	-0.091