

Problem Set #16-Key

Sonoma State University
Economics 305-Intermediate Microeconomic Theory

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Suppose that a market is composed of two firms, each with the same cost structure in which $MC = ATC = \$20$

Total Market Demand is given by: $Q^D = 200 - 2P$.

Assume that the firms behave as Cournot duopolist.

(1) Derive the reaction functions for each firm.

Since the market consist of two firms, $Q^D = 200 - 2P$ can be rewritten as $q_1 + q_2 = 200 - 2P$.
For firm one output can be written as $q_1 = 200 - 2P - q_2$.

For firm one's total revenue function solve the above for P which gives $P = 100 - q_1/2 - q_2/2$.

$$TR_1 = Pq_1 = 100q_1 - q_1^2/2 - q_1q_2/2$$

$$MR_1 = 100 - q_1 - q_2/2.$$

Firm one sets $MR = MC$, to get

$$100 - q_1 - q_2/2 = 20 \text{ solving for } q_1 \text{ gives}$$

$q_1 = 80 - q_2/2$ which is firm one's reaction function or best response function. Since the firms are identical

$$q_2 = 80 - q_1/2 \text{ which is firm two's reaction function or best response function.}$$

(2) Derive the profit maximizing output of each firm.

Since both firms choose their output simultaneously solve the two reaction functions simultaneously by substitution.

$$q_1 = 80 - 1/2 (80 - q_1/2) \text{ solving for } q_1 \text{ produces}$$

$$q_1 = 53.33. \text{ Since both firms are identical,}$$

$$q_2 = 53.33.$$

(3) Derive the profit maximizing price.

Substituting the equilibrium output into the demand curve $P = 100 - q_1/2 - q_2/2$ gives $P = 46.67$.

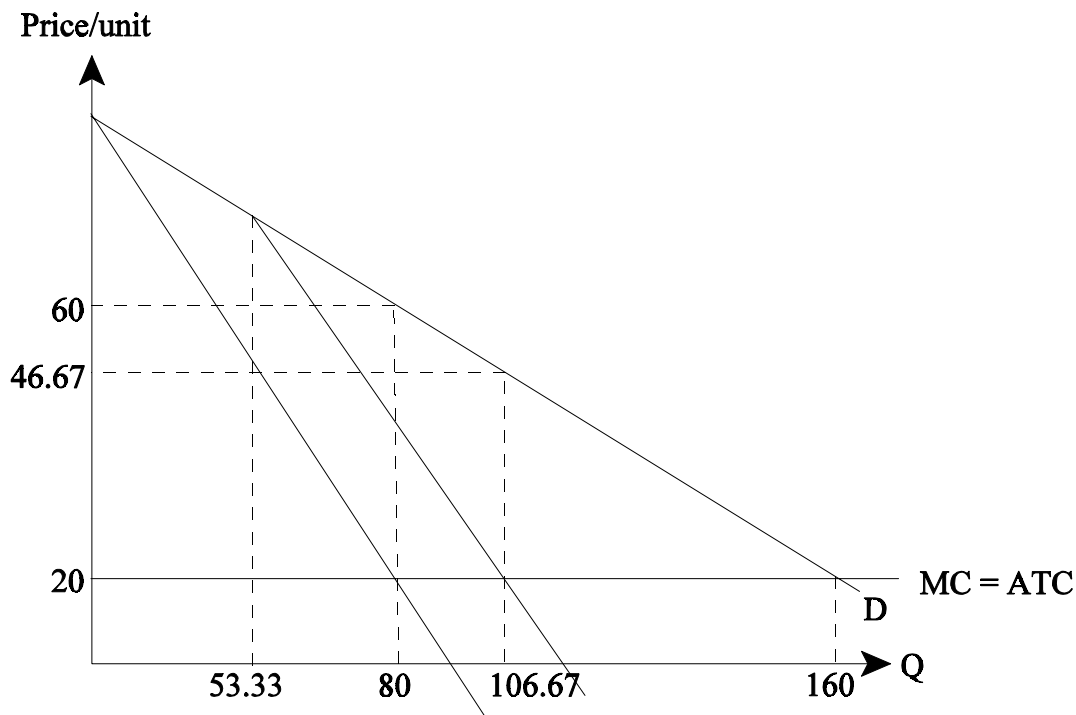
(4) What is the profit per unit at the profit maximizing output?

$$\pi_i/q_i = P - ATC = 46.67 - 20 = \$26.67.$$

(5) What is the total profits of each firm?

$$\pi_i = 26.67(53.33) = \$1,422.31$$

Cournot Model



(6) Show the Cournot equilibrium graphically.

Suppose instead that the firms decide to collude and form a cartel.

(7) What is the profit maximizing total market output? Explain your answer.
The profit maximizing output for a cartel is $Q = 80$.

(8) What is the profit maximizing price?
The profit maximizing price for a cartel is $P = \$60$.

(9) What is the profit per unit at the profit maximizing output?
 $\pi_i/q_i = P - ATC = 60 - 20 = \40 .

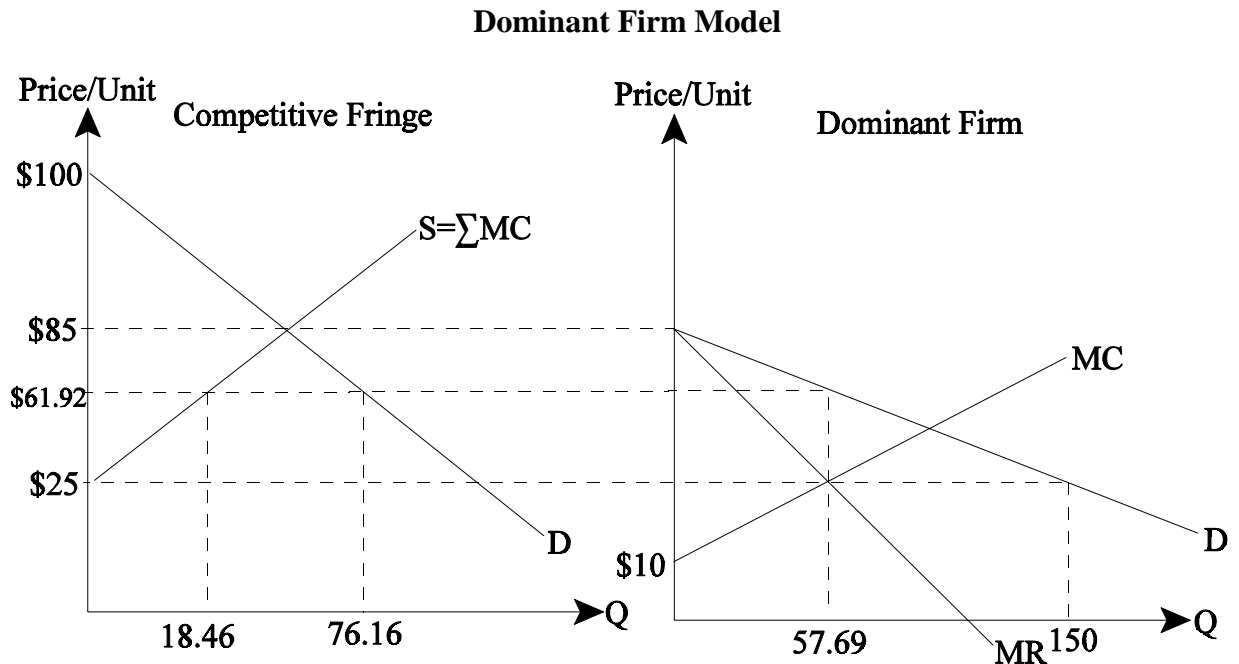
(10) Assuming that the firms equally share the market, what is the total profit of each firm?
 $\pi_i = 40(40) = \$1,600$.

Total Market Demand: $Q^D = 200 - 2P$.

The competitive fringe supply function: $Q^S = \frac{1}{2}P - 12.5$.

The dominant firms marginal cost function: $MC = 10 + \frac{1}{2}Q$.

Use the dominant firm model to answer the following questions.



- (11) What is the minimum price needed for the competitive fringe to supply positive units of output?

The competitive fringe will only supply output at a price above \$25 per unit.

- (12) At what price does the competitive fringe supply output to the entire market?

The competitive fringe will supply the entire market at a price of \$85 and above.

- (13) Derive the dominant firms residual demand function.

The dominant firms demand curve is derived by finding the difference in the quantity supplied by the competitive fringe and the total market demand at prices of \$85 and below. Assuming a linear demand curve, we need two points to derive the equation for the residual demand curve.

$$P_1Q_1 = (85, 0)$$

At a price of \$25 per unit, the dominant firms demand is equal to the market demand so that $Q = 200 - 2(25) = 150$,

$$P_2Q_2 = (25, 150)$$

The general form for a linear demand curve is $Q^D = a - bP$

$$\text{where } b = \frac{\Delta Q}{\Delta P} = \frac{Q_2 - Q_1}{P_2 - P_1} = \frac{150 - 0}{25 - 85} = \frac{150}{-60} = 2.5$$

substitute b into the general form along with one point to get a. Using $P_1Q_1 = (85,0)$

$$0 = a - 2.5(85) \text{ gives } a = 212.5$$

$$Q^D = 212.5 - 2.5P$$

- (14) Derive the dominant firm's marginal revenue function.

Solve the dominant firm's residual demand curve for P and multiply by Q to get the total revenue function.

$$P = 85 - .4Q$$

$$TR = 85Q - .4Q^2$$

$$MR = 85 - .8Q$$

- (15) What is the equilibrium price set by the dominant firm?

Set $MR = MC$

$$85 - .8Q = 10 + \frac{1}{2}Q$$

$$Q = 57.69$$

$$P = 85 - .4(57.69)$$

$$P = \$61.92$$

- (16) At the equilibrium price set by the dominant firm, what is the total market demand?

$$Q^D = 200 - 2(61.92)$$

$$Q = 76.16$$

- (17) At the equilibrium price set by the dominant firm, how much will the competitive fringe supply to the market?

$$Q = \frac{1}{2}(61.92) - 12.5$$

$$Q = 18.46$$

- (18) At the equilibrium price set by the dominant firm, how much will the dominant firm supply to the market?

The dominant firm's output is set in part (5), at $Q = 57.69$.

- (19) Show the above answers graphically. Shown above.

- (20) The competitive fringe reduces the market power of the dominant firm. If the dominant firm wanted to try and eliminate the competitive fringe, how might the dominant firm accomplish this?

The dominant firm can attempt to price below the marginal cost of the competitive fringe. At a price of \$25 or below, the competitive fringe will supply no output and may leave the market.

- (21) If the dominant firm eliminates the competitive fringe, explain how this will effect the model.

The dominant firm's demand curve will become the market demand curve and will effectively become a monopolist.

- (22)

Figure 2

		Firm A	
		Collude	Cheat
Firm B	Collude	(A, B) (20,20)	(A, B) (30,5)
	Cheat	(A, B) (5,30)	(A, B) (10,10)

Suppose that if the two firms collude, each firm will earn twenty million dollars in positive profits. However if one firm cheats on the collusive agreement, the cheating firm will earn thirty million in profits while the abiding firm will earn only five million. If both firms cheat, profits will return to the competitive level of ten million dollars for each firm.

Figure 2 shows the payoff matrix for the two firms resulting from the collusive arrangement.

- (23) Using figure two, construct a payoff matrix illustrating the above outcomes.

Shown above.

- (24) Derive the dominant strategy Nash equilibrium. Explain your answer.

Firm A's best action given what firm B does:

If firm b colludes, firm a should cheat.

If firm b cheats, firm a should cheat.

Firm B's best action given what firm A does:

If firm a colludes, firm b should cheat.

If firm a cheats, firm b should cheat.

- (25) Is there a solution that would benefit both firms more than the dominant strategy Nash equilibrium? If so, explain why this is not the Nash equilibrium.

Yes, if both firms collude, each would be better off.

- (26) Why is the collusive arrangement unstable?

The collusive arrangement is unstable because the high profits resulting from collusion create an incentive to increase production beyond the collusive level. As each firm increases output, price and profits return to the competitive level.