

Problem Set #7-Key

Sonoma State University
Economics 305-Intermediate Microeconomic Theory

Dr. Cuellar

Consider a firm that faces the following total product curves depending on how much capital it employs.

K=1 Unit			K=2 Units			K=3 Units		
Quantity of Labor	Total Product	MP _L	Quantity of Labor	Total Product	MP _L	Quantity of Labor	Total Product	MP _L
1	100	-----	1	123	-----	1	139	-----
2	152	52	2	187	64	2	193	54
3	193	41	3	237	50	3	263	70
4	215	22	4	263	26	4	319	56
5	233	18	5	286	23	5	366	47
6	249	16	6	306	20	6	407	41
7	263	14	7	323	17	7	410	3

- (1) Compute the marginal product of labor (MP_L) in the table above.
- (2) Suppose the firm currently employs 1 unit of capital and 3 units of labor.
- (i) Compute the MP_L and the MP_K.
MP_K = ΔTP/ΔK holding labor fixed at 3 units.

$$\frac{\Delta TP}{\Delta K} = \frac{193 - 0}{1} = 193$$

- (ii) Compute the MRTS_{LK}.
MRTS_{LK} = MP_L / MP_K
= 41/193
= .212

- (3) Suppose that the firm currently employs 2 units of capital. The price of capital is \$4 per unit and the price of labor is \$10 per unit.
- (i) What is the short run total cost of producing 263 units?
TC = wL + rK
= \$10(4) + \$4(2)
TC = \$48

- (ii) What is the long run total cost of producing 263 units?
To see if the costs of producing 263 units can be reduced by reallocating resources, you need to check your cost minimizing condition, i.e., does MP_L/MP_K = w/r.

$$\frac{MP_L}{MP_K} = \frac{26}{48} = .542, \quad \frac{w}{r} = \frac{10}{4} = 2.5$$

Clearly $.542 < 2.5$, so that the firm is not minimizing costs. This firm should use more capital and less labor.

If the firm used three units of labor and 3 units of capital the firm can reduce costs to,
 $TC = \$10(3) + \$4(3) = \$43$

- (4) Suppose again that the firm currently employs two units of capital but that the price of capital increases to \$20 per unit and the price of labor falls to \$5 per unit.

(i) What is the short run total cost of producing 263 units?

$$TC = \$5(4) + \$20(2) = \$60$$

(ii) What is the long run total cost of producing 263 units?

The marginal products of labor and capital stay the same so that $MRTS = .542$. However, the factor price ratio changes to $5/20 = .25$ so that $MRTS > w/r$. The firm can now reduce costs by using more labor and less capital.

If the firm uses seven units of labor and one unit of capital the firm can reduce costs to,
 $TC = \$5(7) + \$20(1) = \$55$

- (5) Beginning with one unit of capital and 2 units of labor, does this production function exhibit increasing, constant or decreasing returns to scale?

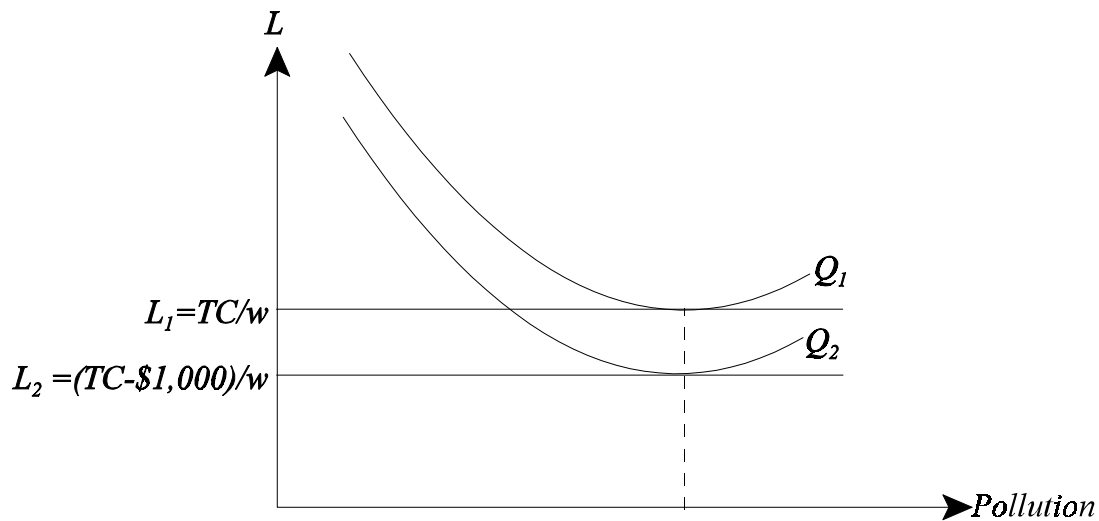
To examine returns to scale double both inputs and see what happens to output.

With two units of capital and four units of labor output increases from 152 to 263. Output increases by less than double (73%) so the firm experiences decreasing returns to scale.

- (6) Prove that when resources are allocated such that $\frac{MP_L}{w} = \frac{MP_K}{r}$, costs are minimized (or alternatively output is maximized).
Note, your answer should include a discussion of total product, marginal product and the law of diminishing marginal returns.

Prove by contradiction.

- (7) Suppose that a firm producing Kathy Lee Gifford CD's employs only two inputs, labor and toxic emissions. It turns out the production of Kathy Lee CD's somehow results in toxic fumes emitted into the atmosphere. The firm faces a positive wage rate for labor but faces no charge for emitting toxic fumes.
- (a) Given a fixed total cost of production, show graphically using iso-cost and iso-quant curves, the cost minimizing output. Explain fully the equilibrium condition that holds at the cost minimizing output and the implications of this condition on factor usage.

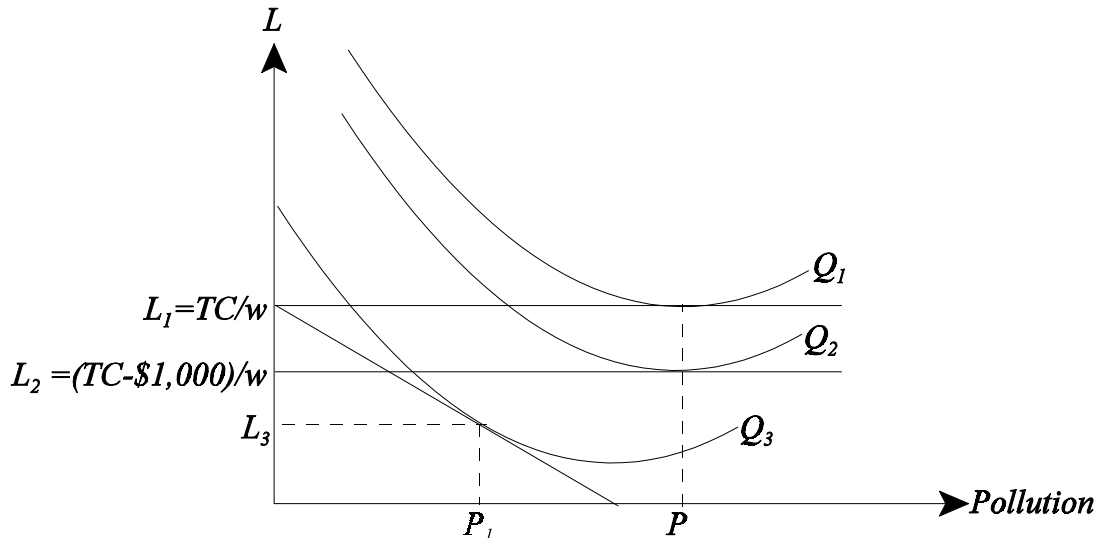


At the equilibrium $MRTS = r/w = 0$. Since $MRTS = MP_p/MP_L$, this implies that pollution is used until $MP_p = 0$. That is, pollution is used until it is no longer productive.

- (b) Suppose that in an effort to reduce air pollution, the government imposes a fixed annual fee of \$1,000 on the firm. Show graphically using iso-cost and iso-quant curves how this will affect the equilibrium level of output and the amount of toxic fumes emitted by the firm. Explain your answer.

The iso-cost line shifts down by the amount of the fee, reducing labor usage but keeping pollution emissions the same.

- (c) Suppose that instead of fixed annual fee, a per unit fee for toxic emissions is charged. Show graphically using iso-cost and iso-quant curves how this will affect the equilibrium level of output and the amount of toxic fumes emitted by the firm. Explain your answer.



Facing a positive per unit fee on pollution, the firm economizes on pollution use and reduces emissions from P to P_1 .

- (d) Suppose the above firm hires an economist to estimate its production function. Through econometric analysis, the production function is estimated to be $Q = 6LP$, where Q = annual number of Kathy Lee Gifford CD's produced, L = labor hours, and P = tons of pollution emitted into the air. The marginal products of labor and pollution are $MP_L = 6P$ and $MP_P = 6L$ respectively. Assume the wage rate faced by the firm is \$7.50 per hour and the per unit emissions fee is \$30 per ton. If the firm's budget is \$300,000, how much labor and pollution should the firm employ? How much output will the firm produce?

Using the equilibrium condition $MRTS = r/w$ and the fact that $MRTS = MP_P/MP_L$, we get,

$$MP_P / MP_L = r/w \Rightarrow 6L/6P = 30/7.50. \text{ Solving for } L \text{ we get, } L = (30/7.50)P$$

Substituting L into the cost function $TC = wL + rP$ and using the data given, gives

$$300,000 = 7.50(30/7.50)P + 30P$$

$$300,000 = 30P + 30P$$

$$300,000 = 60P$$

$$P = 5,000 \text{ tons of pollution}$$

$$L = (30/7.50)(5000) = 20,000 \text{ labor hours.}$$

$$Q = 6(20,000)(5,000) = 800,000,000 \text{ Kathy Lee Gifford cd's produced. God help us all.}$$