Example A

Find the Fourier Series Coef., C_k :

$x(t) = \cos(2t + \pi/4)$

Fundamental frequency:
$w_0 = 2 = \frac{2\pi}{T_0} \Rightarrow T_0 = \pi$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jw_0 t k}$$

$$= \cos(2t + \pi/4) \quad \text{using Euler's}$$

$$= \tfrac{1}{2} e^{j(2t+\pi/4)} + \tfrac{1}{2} e^{-j(2t+\pi/4)}$$

$$= \tfrac{1}{2} e^{j\pi/4} \underbrace{\left(e^{j2t}\right)}_{k=1} + \tfrac{1}{2} e^{-j\pi/4} \underbrace{\left(e^{-j2t}\right)}_{k=-1}$$

$\Rightarrow$ $C_k \neq 0$ for $k=1$ & $k=-1$ or $|k| = 1$

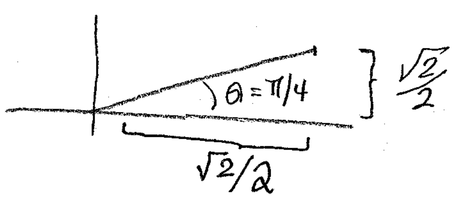
Polar

$\Rightarrow$ $k = 1$ $\quad C_k = C_1 = \tfrac{1}{2} e^{j\pi/4} \Rightarrow \boxed{|C_1| = \tfrac{1}{2} \quad \theta_{C_1} = \pi/4}$

or $\tfrac{1}{2}\left(\frac{\sqrt{2}}{2}\right)[1 + j] = \boxed{\frac{\sqrt{2}}{4}(1 + j)}$

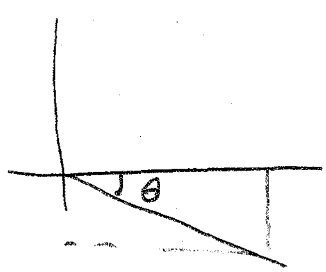
Rectangular

remember:



$k = -1$ $\quad C_k = C_{-1} = \tfrac{1}{2} e^{-j\pi/4}$ same as above but negative angle $\Rightarrow$ Negative Im. part

$\boxed{|C_{-1}| = \tfrac{1}{2} \quad \theta_{C_{-1}} = -\pi/4}$



or

$\boxed{\frac{\sqrt{2}}{4}(1 - j)}$

Example B

Find the Fourier Series Coef. of

$$x(t) = \sin^2 t$$

Note $\quad \sin t = \frac{1}{2j} e^{jt} - \frac{1}{2j} e^{-jt}$

Remember:

$$\frac{1}{2j} = -\frac{j}{2}$$

$$(j)^2 = -1, \quad (j)^3 = (-j)$$

$$(-j)^2 = (-1)^2 j^2 = -1$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\omega_0 t k} \qquad \omega_0 = 1 = \frac{2\pi}{T_0} \rightarrow T_0 = 2\pi$$

$$= \sin^2 t = \left[\frac{1}{2j} e^{jt} - \frac{1}{2j} e^{-jt}\right]^2$$

$$= \left(\frac{1}{2j}\right)^2 \left(e^{jt}\right)^2 + \left(\frac{-1}{2j}\right)^2 \left(e^{-jt}\right)^2 - 2\left(\frac{1}{2j}\right) e^{jt} \cdot \left(\frac{+1}{2j}\right) e^{-jt}$$

$$= \frac{(-j)^2}{4} e^{j2t} + \frac{j^2}{4} e^{-j2t} - \frac{2}{4(j \cdot j)} \cancel{e^{jt} \cdot e^{-jt}}_{1}$$

$$= \underbrace{\frac{-1}{4} e^{j2t}}_{k=1} + \underbrace{\frac{-1}{4} e^{-j2t}}_{k=-1} + \underbrace{\frac{+1}{2}}_{k=0}$$

$$\Rightarrow \underbrace{C_1 = -1/4}_{\text{Real}} \qquad \underbrace{C_{-1} = -1/4}_{\text{Real}} \qquad \underbrace{C_0 = +1/2}_{\text{DC value}}$$

$\theta_{C_1} = 0 \qquad \theta_{C_{-1}} = 0 \qquad$ Never has an phase angle

$$\Rightarrow C_k = 0 \quad |k| \neq 1, 0$$

Example 6

Find C_k ; what is ω_0 ?

$$x(t) = \cos 4t + \sin 6t$$

Using Euler's formula :

$$x(t) = \tfrac{1}{2} e^{j4t} + \tfrac{1}{2} e^{-j4t} + \frac{1}{2j} e^{j6t} - \frac{1}{2j} e^{-j6t}$$

$$= \sum_{k=-\infty}^{\infty} C_k e^{-j\omega_0 t k}$$

Note ; We have to find The fundamenta frequency (ω_0)

Consider $e^{j\omega_0 t k} \equiv e^{j4t} \; \& \; e^{j6t} \rightarrow e^{j2t} \begin{cases} k=2 & e^{j4t} \\ k=3 & e^{j6t} \end{cases}$

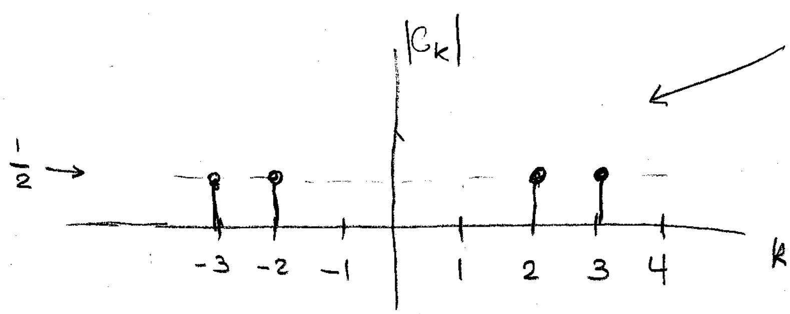
$$\Rightarrow \omega_0 = 2 = \frac{2\pi}{T_0} \Rightarrow T_0 = \pi$$

$$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} C_k e^{-j2t(k)} = \underset{k=2}{\tfrac{1}{2} e^{j2t(2)}} + \underset{k=-2}{\tfrac{1}{2} e^{-j2t(2)}} + \underset{k=3}{\frac{1}{2j} e^{j2t(3)}} - \underset{k=-3}{\frac{1}{2j} e^{-j2t(3)}}$$

$$\left.\begin{aligned} C_2 &= +\tfrac{1}{2} &&= \tfrac{1}{2} \\ C_{-2} &= \tfrac{1}{2} &&= \tfrac{1}{2} \\ C_{+3} &= \tfrac{1}{2j} &&= \tfrac{1}{2} e^{-j\pi/2} \\ C_{-3} &= \tfrac{-1}{2j} &&= \tfrac{1}{2} e^{+j\pi/2} \\ C_k &= 0 \quad |k| \neq 2 \,\&\, 3 \end{aligned}\right\}$$

Fourier Coefficients of $x(t)$

Has no DC value

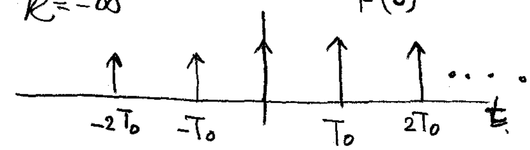


we plot the magnitude of $X(t)$'s freq. spectrum

Example D

Assume $p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_o)$

This is an impulse train



Find Fourier series Coefficients of $x(t)$.

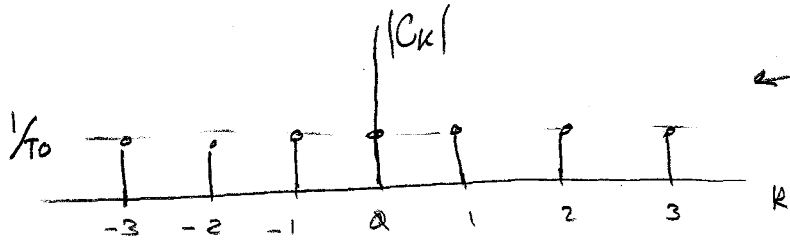
$$P(t) = \sum_{k=-\infty}^{\infty} C_k e^{jw_otk}$$ but what is $C_k = ?$

$$C_k = \frac{1}{T_o}\int_{T_o} P(t) e^{-jw_ot} dt$$, note $P(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_o)$ has value $\equiv \delta(t)$ at every k

$$= \frac{1}{T_o}\int_{T_o} \delta(t) e^{-jw_ot} dt$$ using $\int f(t)\delta(t - t_o)dt$ to

$$\Rightarrow \frac{1}{T_o} e^{-jw_o(0)} = \frac{1}{T_o}$$

$$\Rightarrow C_k = \frac{1}{T_o} \quad \forall k \quad (\text{For all } k\text{s})$$



mag. of the freq. Spectrum

→ what happens as T_o changes?
→ what is the DC value?

Look at Table 4.3 (P. 170 3rd Ed) Case 7. In this case $X_o = 1$