Math 220 **Sample Final** **8 December 2004**

(with thanks to Dr. B)

1. Complete each of the following definitions.

   (a) $f : D \to C$ is a **one-to-one function** if ____________________________

   (b) $f : D \to C$ is an **onto function** if ____________________________

   (c) Assume $f : D \to C$ and $B \subseteq C$. $f^{-1}[B] = \{$____________________________$\}$

   (d) X is **equinumerous to** Y ($X \approx Y$) if ____________________________

   (e) A is **denumerable** if ____________________________

   (f) Assume T is a relation on A. T is **reflexive** on A if ____________________________

   (g) Assume X is a poset and $A \subseteq X$. $x = \text{lub}(A)$ if ____________________________

2. (a) Give the **converse** of $x \notin A \Rightarrow x \in B$.

   (b) Give the **contrapositive** of $x \notin A \Rightarrow x \in B$.

3. (a) Give the **negation** of $\forall_{y \geq 0}(\forall_{\varepsilon > 0}(y < \varepsilon) \Rightarrow y = 0)$.

   (b) Is the negation or the original statement true in part (a)?

4. Assume that $A \subseteq B$. Use the pick-a-point method for parts (a) and (b).

   (a) Prove $A \cup B = B$.

   (b) Let $f : D \to C$, and assume A, B are subsets of C. Prove $f^{-1}[A] \subseteq f^{-1}[B]$.

5. Use mathematical induction to prove that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n = \frac{(n-1)n(n+1)}{3} \text{ for } n = 2, 3, 4, \ldots$$

6. Define a relation S on the interval $[-\pi, \pi]$ by: xSy if $x - y$ is rational. [You may assume 1) $a \pm b$ is rational if a and b are both rational, and 2) $a \pm b$ is irrational if exactly one of a and b is irrational.]

   (a) Prove S is an equivalence relation.

   (b) Describe the equivalence class $[0]$. In other words, what is in $[0]$?

   (c) Is $3 - e \in [e]$? Justify your answer.

You should also know the definitions for a **ring, ring homomorphism, kernel** of a homomorphism, and **isomorphism**.

Review exercises for chapter 8: p. 258, # 1, 2, 3, 4, 5, 6, 11, 13, 15, 16.