

QUINT-PRIMES INVESTIGATION

THE PROBLEM

Quints Suppose Q is the set of all multiples of five [by natural numbers], as well as 1. So $Q = \{1, 5, 10, 15, 20, \dots\}$. In the set Q , there are some numbers that can only be written as products of 1 and the number itself, and not as the product of any other combination of elements in the set. So $10 = 1 \times 10$ is such a number, since the usual factoring of 2×5 involves a number that is not in Q . Call such numbers Q -primes.

Other numbers in Q have more than one factorization into elements of Q . One such number is 25, since $25 = 1 \times 25$ and also $25 = 5 \times 5$. Call these numbers Q -composites.

The number 1 is considered special, neither Q -prime nor Q -composite, just as it is in our usual natural number system.

Answer the following questions (1 – 4).

1. Determine the first 20 Quint-primes. Would the sequence of Quint-primes continue indefinitely? Why? Describe any patterns you find relating to the Quint-primes. Describe a specific test (algorithm) to determine whether a given larger number in Q is Quint-prime or not.
2. Is the set Q closed under multiplication? Explain.
3. Is there a “Fundamental Theorem of Arithmetic” that applies to set Q ? In other words, can every Quint-composite in Q be factored into a product of Quint-primes, and is every such factorization unique? Explain.
4. How are Quint-primes like primes in the natural numbers? How are they different?

Now consider the following sets, and for each one, assume that set-primes and set-composites are defined in a way comparable to the definitions of Q -primes and Q -composites. Answer questions 1, 2, 3, and 4 above for each set.

Odd-primes. $O = \{1, 3, 5, 7, 9, \dots\}$, the set of odd numbers.

Even-primes. $E = \{1, 2, 4, 6, 8, 10, \dots\}$, the set of even numbers together with 1.

UnThree-primes. $U = \{1, 2, 4, 5, 7, 8, 10, 11, \dots\}$, the set of natural numbers that are not multiples of 3.

NotFour-primes. $N = \{1, 2, 3, 5, 6, 7, 9, 10, \dots\}$, the set of natural numbers that are not multiples of 4.

Leight-primes. $L = \{1, 8, 16, 24, 32, 40, \dots\}$, the set of multiples of 8 together with 1.

Finally, look for more general patterns and do your best to describe why these patterns hold.

5. Are there any patterns that hold for all six sets and their primes? Explain what these are and why they hold.
6. Try to find groups of sets and their primes that share similar patterns and properties. Describe the common properties of each group, and explain why they hold. Generalize the types of sets which have these properties.

PRESENTATION

Write a single group report which:

- states the problem and describes in general how your group went about solving it; and
- answers each question clearly, using appropriate tools and methods of mathematical communication.

Your paper should include enough examples and explanation that the reader can follow your reasoning clearly. The report can take any form or length you see fit, but it must address the problem adequately and be well-written and neat. Tables and mathematical work may be neatly hand-written, but all narrative should be word-processed.