

Expectations:

In addition to finding all legitimate answers you need to justify your solution[s], explaining in detail how you found the answers and why they work. Getting correct answers is certainly important, but understanding the reasoning processes involved and the sense of the problem and its solution are even more important. Your take home exam will be evaluated using all of these criteria.

A correct answer with no supporting explanation will receive no credit. A reasonable but incorrect or incomplete answer together with the calculations that produced it will receive partial credit. In order to receive full credit, you must:

- state the problem clearly, preferably in your own words;
- explain how you went about solving the problem, including any assumptions you may have made;
- show any examples, data, or calculations you used, in a clear and organized way (this may include diagrams, lists, tables, graphs, or other visual displays);
- state complete and correct answer[s];
- explain why this is a reasonable solution to the problem.

Your work should be typed or word-processed, though calculations, diagrams, tables or other graphic displays may be hand-drawn. I remind you that: You must write up your own take-home quizzes and exams and homework assignments **independently without help**, but you may collaborate on all problems and ideas in preparation for that writing.

1. Suppose  $a$ ,  $b$ , and  $c$  stand for any positive integers. Decide whether each of the following statements is true or false. Recall that true must be ALWAYS true, so even one counterexample is enough to answer false. If the statement is true, give a general argument [beyond just specific examples] why it is true. If the answer is false, give a counterexample. A factor does not have to be prime.
  - a) If  $a$  is a factor of the product of  $b$  and  $c$ , then  $a$  is a factor of  $b$  and  $a$  is a factor of  $c$ .
  - b) If  $a$  is a factor of  $b$  and  $a$  is not a factor of  $c$ , then the product of  $b$  and  $c$  is a multiple of  $a$ .
  - c) If  $a$  is a factor of  $b$  and  $a$  is a factor of the product of  $b$  and  $c$ , then  $a$  is a factor of  $c$ .
  - d) If  $a$  is not a factor of  $b$ , then  $b^2$  is not a multiple of  $a$ .
  - e) If  $a$  is not a factor of  $b$  and  $a$  is not a factor of  $c$ , then  $a$  is not a factor of the product of  $b$  and  $c$ .
  
2. Draw appropriate diagram(s) to solve this word problem. The diagram(s) must be used to solve the problem, not just to illustrate the solution. Explain each step in your reasoning, including your reasons for setting up your diagram(s) in the manner you chose.
 

“An automobile company had its budget slashed by one-fifth this fiscal year. That is, the total budget for this year was cut by  $\frac{1}{5}$  from last year’s budget. Each year,  $\frac{3}{5}$  of the total budget goes for direct manufacturing costs. This year’s budget for manufacturing costs is \$72,000,000. What was the total budget last year?”
  
3. Enter the following sequence of numbers and instructions on a calculator. The values that should be calculated by the machine are given in bold. (These values may appear slightly different on your calculator due to rounding variations. Any rounding variations are not the point of this problem.)
 
$$146 \div 17 = \mathbf{8.588235}$$

$$8.588235 - 8 = \mathbf{.5882352}$$

$$.5882352 \times 17 = \mathbf{10}$$
  - a) Explain what meaningful results this calculation yields — i.e., what does **10** mean in this problem?
  - b) Repeat this same algorithm [changing the numbers, of course] with  $238 \div 9$  and with  $6053 \div 4$ . Record the steps as in the example above. State what you found in each case.
  - c) Describe this algorithm in words as a series of steps for dividing two natural numbers. E.g.: “Step 1. Enter the dividend.”
  - d) Explain what this algorithm does and why it works.