Properties of Fourier Transform

1	*Time*	*Frequency*
2	$\alpha_1 x_1(t) \pm \alpha_2 x_2(t)$	$\alpha_1 X_1(j\omega) \pm \alpha_2 X_2(j\omega)$
3	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
4	$x(at)$	$\frac{1}{\|a\|} X\left(\frac{j\omega}{a}\right)$
2&3	$x(at - t_0)$	$\frac{1}{\|a\|} e^{-j\left(\frac{\omega}{a}\right)t_0} X\left(\frac{j\omega}{a}\right)$
4	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n} X(j\omega)$
5	$x(t)e^{j\omega_0 t}$	$X(j(\omega - \omega_0))$
6	$x(t)\cos(\omega_0 t)$ $x(t)\sin(\omega_0 t)$	$\frac{1}{2}[X(j(\omega + \omega_0)) + X(j(\omega - \omega_0))]$ $\frac{j}{2}[X(j(\omega + \omega_0)) - X(j(\omega - \omega_0))]$
7	$\frac{d^n}{dt^n} x(t)$	$(j\omega)^n X(j\omega)$
7a	$(-jt)^n x(t)$	$\frac{d^n X(j\omega)}{d\omega^n}$
8	$x_1(t) * x_2(t)$	$X_1(j\omega) X_2(j\omega)$
9	$x_1(t) x_2(t)$	$\frac{1}{2\pi} X_1(j\omega) * X_2(j\omega)$
10	$x(-t)$	$X(-j\omega)$
11	$X(jt)$	$2\pi x(-\omega)$
11a	$X(-jt)$	$2\pi x(\omega)$
12	$\int_{-\infty}^{t} x(\tau) d\tau$	$\frac{1}{j\omega} X(j\omega) + \pi X(j0)\delta(\omega)$
Parseval's Theorem 13	$\int_{-\infty}^{\infty} \|x(t)\|^2 dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} \|X(j\omega)\|^2 d\omega$

Common Fourier Transform Pairs

	Time	*Frequency*
1	$\delta(t)$	1
2	$e^{-at}u_h(t), \quad a>0$	$\frac{1}{a+j\omega}$
3	$p^h_\tau(t)$	$\tau \text{sinc}(\frac{\omega\tau}{2\pi}) = \tau \frac{\sin(\omega\tau/2)}{\omega\tau/2}$
4	$\Delta_\tau(t)$	$\frac{\tau}{2}\text{sinc}^2(\frac{\omega\tau}{4\pi})$
5	1	$2\pi\delta(\omega)$
6	const	const $\times\, 2\pi\delta(\omega)$
7	$\cos(\omega_0 t)$	$\pi[\delta(\omega+\omega_0)+\delta(\omega-\omega_0)]$
8	$\sin(\omega_0 t)$	$j\pi[\delta(\omega+\omega_0)-\delta(\omega-\omega_0)]$
9	$\text{sgn}(t)$	$\frac{2}{j\omega}$
10	$u_h(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
11	$e^{-\alpha\|t\|}, \quad \alpha>0$	$\frac{2\alpha}{\alpha^2+\omega^2}$
12	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$

13	$\sum_{n=-\infty}^{\infty} X_n e^{jn\omega_0 t}, \quad \omega_0 = \frac{2\pi}{T}$	$2\pi \sum_{n=-\infty}^{\infty} X_n \delta(\omega - n\omega_0)$
14	$\sum_{n=-\infty}^{\infty} \delta(t-nT_0)$	$\omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_0), \quad \omega_0 = \frac{2\pi}{T_0}$

Properties of Laplace Transform

1	$\mathcal{L}\{\alpha_1 f_1(t) \pm \alpha_2 f_2(t)\}$	$\alpha_1 F_1(s) \pm \alpha_2 F_2(s)$
2	$\mathcal{L}\{f(t-t_0)u(t-t_0)\}$	$e^{-st_0}F(s), \quad t_0 > 0$
3	$\mathcal{L}\{f(at)\}$	$\frac{1}{a}F(\frac{s}{a}), \quad a > 0$
4	$\mathcal{L}\{t^n f(t)\}$	$(-1)^n \frac{d^n}{ds^n}F(s)$
5	$e^{\lambda t} f(t)$	$F(s-\lambda)$
6	$f(t)\cos(\omega_0 t)$ $f(t)\sin(\omega_0 t)$	$\frac{1}{2}[F(s+j\omega_0)+F(s-j\omega_0)]$ $\frac{j}{2}[F(s+j\omega_0)-F(s-j\omega_0)]$
7	$\mathcal{L}\left\{\frac{d}{dt}f(t)\right\}$	$sF(s) - f(0^-)$
8	$\mathcal{L}\left\{\frac{d^2}{dt^2}f(t)\right\}$	$s^2F(s) - sf(0^-) - f^{(1)}(0^-)$

9	$\mathcal{L}\left\{\frac{d^n}{dt^n}f(t)\right\}$	$s^nF(s) - s^{n-1}f(0^-) - s^{n-2}f^{(1)}(0^-)$ $- \cdots - f^{(n-1)}(0^-)$
10	$\mathcal{L}\{f_1(t) * f_2(t)\}$	$F_1(s)F_2(s)$
11	$\mathcal{L}\{\int_0^t f(\tau)d\tau\}$	$\frac{1}{s}F(s)$
12	$\lim_{t\to 0^+}\{f(t)\}$	$\lim_{s\to\infty}\{sF(s)\}$
13	$\lim_{t\to\infty}\{f(t)\}$	$\lim_{s\to 0}\{sF(s)\}$

Common Laplace Transform Pairs

1	$\delta(t)$	1
2	$u(t)$	$\frac{1}{s}$
3	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$
5	$t^n e^{-\alpha t}u(t)$	$\frac{n!}{(s+\alpha)^{n+1}}$
6	$u(t)\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
7	$u(t)\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
8	$e^{-\alpha t}u(t)\cos(\omega t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega^2}$
9	$e^{-\alpha t}u(t)\sin(\omega t)$	$\frac{\omega}{(s+\alpha)^2+\omega^2}$
10	$tu(t)\cos(\omega t)$	$\frac{s^2-\omega^2}{(s^2+\omega^2)^2}$
11	$tu(t)\sin(\omega t)$	$\frac{2\omega s}{(s^2+\omega^2)^2}$
12	$te^{-\alpha t}u(t)\cos(\omega t)$	$\frac{(s+\alpha)^2-\omega^2}{((s+\alpha)^2+\omega^2)^2}$
13	$te^{-\alpha t}u(t)\sin(\omega t)$	$\frac{2\omega(s+\alpha)}{((s+\alpha)^2+\omega^2)^2}$

Source: Zoran Gajic, “Linear Dynamic Systems and Signals”

Properties of Z Transform

$\mathcal{Z}\{a_1 f_1[k] + a_2 f_2[k]\}$	$a_1 F_1(z) + a_2 F_2(z)$	1
$\mathcal{Z}\{f[k-k_0]u[k-k_0]\}$	$\frac{1}{z^{k_0}} F(z)$	2
$\mathcal{Z}\{f[k-1]u[k]\}$	$\frac{1}{z} F(z) + f[-1]$	3
$\mathcal{Z}\{f[k-2]u[k]\}$	$\frac{1}{z^2} F(z) + \frac{1}{z} f[-1] + f[-2]$	4
$\mathcal{Z}\{f[k-k_0]u[k]\}$	$\frac{1}{z^{k_0}} F(z) + \frac{1}{z^{k_0-1}} f[-1] + \cdots + \frac{1}{z} f[-k_0+1] + f[-k_0]$	5
$\mathcal{Z}\{f[k+1]u[k]\}$	$zF(z) - zf[0]$	6
$\mathcal{Z}\{f[k+2]u[k]\}$	$z^2 F(z) - z^2 f[0] - zf[1]$	7
$\mathcal{Z}\{f[k+k_0]u[k]\}$	$z^{k_0} F(z) - z^{k_0} f[0] - z^{k_0-1} f[1] - \cdots - zf[k_0-1]$	8
$\mathcal{Z}\{kf[k]\}$	$-z \frac{d}{dz} F(z)$	9
$\mathcal{Z}\{k^2 f[k]\}$	$z \frac{d}{dz} F(z) + z^2 \frac{d^2}{dz^2} F(z)$	10
$\mathcal{Z}\{a^k f[k]\}$	$F\left(\frac{z}{a}\right)$	11
$\mathcal{Z}\{f[k]\cos(\omega kT)\}$	$\frac{1}{2}\left[F\left(ze^{j\omega T}\right) + F\left(ze^{-j\omega T}\right)\right]$	12
$\mathcal{Z}\{f[k]\sin(\omega kT)\}$	$\frac{j}{2}\left[F\left(ze^{j\omega T}\right) - F\left(ze^{-j\omega T}\right)\right]$	13
$\mathcal{Z}\{f_1[k] * f_2[k]\}$	$F_1(z) F_2(z)$	14
$\lim_{k \to 0} f[k]$	$\lim_{z \to \infty} \{F(z)\}$	15
$\lim_{k \to \infty} f[k]$	$\lim_{z \to 1} \left\{\frac{z-1}{z} F(z)\right\}$	16

Common Z Transform Pairs

$\delta[k]$	1	1
$u[k]$	$\frac{z}{z-1}$	2
$a^k u[k]$	$\frac{z}{z-a}$	3
$k u[k]$	$\frac{z}{(z-1)^2}$	4
$k^2 u[k]$	$\frac{z(z+1)}{(z-1)^3}$	5
$k a^k u[k]$	$\frac{az}{(z-a)^2}$	6
$k^2 a^k u[k]$	$\frac{az(z+a)}{(z-a)^3}$	7
$\frac{1}{m!}k(k-1)(k-2)\cdots(k-m+1)u[k]$	$\frac{z}{(z-1)^m}$	8
$\frac{1}{m!}k(k+1)(k+2)\cdots(k+m)a^k u[k]$	$\frac{z^{m+1}}{(z-a)^{m+1}}$	9
$u[k]\cos(\omega kT)$	$\frac{z^2-z\cos(\omega T)}{z^2-2z\cos(\omega T)+1}$	10
$u[k]\sin(\omega kT)$	$\frac{z\sin(\omega T)}{z^2-2z\cos(\omega T)+1}$	11
$a^k u[k]\cos(\omega kT)$	$\frac{z^2-az\cos(\omega T)}{z^2-2az\cos(\omega T)+a^2}$	12
$a^k u[k]\sin(\omega kT)$	$\frac{az\sin(\omega T)}{z^2-2az\cos(\omega T)+a^2}$	13

Source: Zoran Gajic, “Linear Dynamic Systems and Signals”

Summary of Fourier Series

Given a complex-valued function *f* of real argument *t, f : R* ➞ *C*, where *f(t)* is piecewise continuous, periodic with period *T*, and square-integrable over the interval from t_1 to t_2 of length *T*, that is,

$$\int_{t_1}^{t_2} |f(t)|^2 dt < +\infty$$

where

- $T = t_2 - t_1$, is the period,
- t_1 and t_2 are times to integrate between.

The **Fourier series expansion** of *f* is:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(\omega_n t) + b_n \sin(\omega_n t)]$$

where, for any non-negative integer *n*:

- $\omega_n = n\frac{2\pi}{T}$ is the *n*th harmonic (in radians) of the function *f*,
- $a_n = \frac{2}{T}\int_{t_1}^{t_2} f(t)\cos(\omega_n t)\, dt$ are the even **Fourier coefficients** of *f*, and
- $b_n = \frac{2}{T}\int_{t_1}^{t_2} f(t)\sin(\omega_n t)\, dt$ are the odd **Fourier coefficients** of *f*.

Equivalently, in exponential form,

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n e^{i\omega_n t}$$

where:

- $c_n = \frac{1}{T}\int_{t_1}^{t_2} f(t)e^{-i\omega_n t}\, dt,$

Source: http://en.wikipedia.org/wiki/Fourier_series