

Math 161 – Sample Exam 1 Problems

Note. This is not a template. It is likely that topics that don't appear here will appear on the in-class exam. However, these problems are intended to give you an idea of the style and difficulty level of exam questions, as well as to remind you of some of the major topics we have covered.

1. Use the **limit definition** of the derivative to calculate $f'(x)$ if $f(x) = \frac{1}{x+2}$. Show all intermediate algebra steps.
2. For each of the following, calculate $\frac{dy}{dx}$. **REMINDER.** A CALCULATOR WILL NOT BE ALLOWED ON THE EXAM FOR A QUESTION LIKE THIS ONE.

(a) $y = 1 - 2x + \frac{5}{\sqrt[3]{x}}$

(b) $y = 2e^x + 3x^e$

(c) $y = \sqrt{x} + \sqrt{2}$

(d) $y = (x + e^x)(3 - \sqrt{x})$

(e) $y = \frac{x^3 + 1}{x^4 - 2}$

3. Calculate each of the following limits. Calculate the first three exactly, and use the table method to approximate the fourth one.

(a) $\lim_{x \rightarrow 4} (x^2 + 2)$

(b) $\lim_{x \rightarrow -3} \frac{x^2 - x - 12}{x + 3}$

(c) $\lim_{h \rightarrow 0} \frac{\sqrt{h+16} - 4}{h}$

(d) $\lim_{x \rightarrow 0^+} x^x$

4. Let

$$f(x) = \begin{cases} 2 - x & \text{if } x < -1 \\ x & \text{if } -1 \leq x < 1 \\ 4 & \text{if } x = 1 \\ 4 - x & \text{if } x > 1 \end{cases}.$$

Calculate each of the following limits exactly, and determine for which points $f(x)$ is continuous.

(a) $\lim_{x \rightarrow -1^-} f(x)$

(b) $\lim_{x \rightarrow -1^+} f(x)$

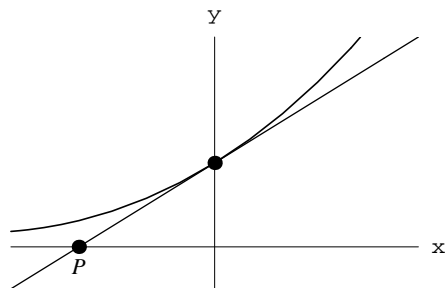
(c) $\lim_{x \rightarrow -1} f(x)$

(d) $\lim_{x \rightarrow 1^-} f(x)$

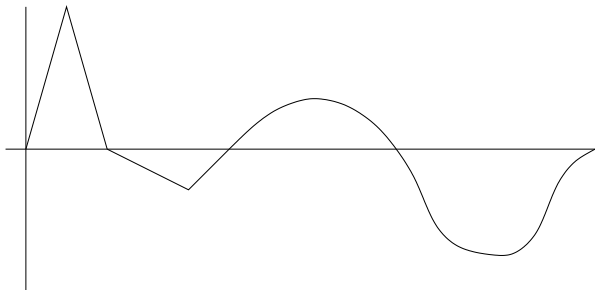
(e) $\lim_{x \rightarrow 1^+} f(x)$

(f) $\lim_{x \rightarrow 1} f(x)$

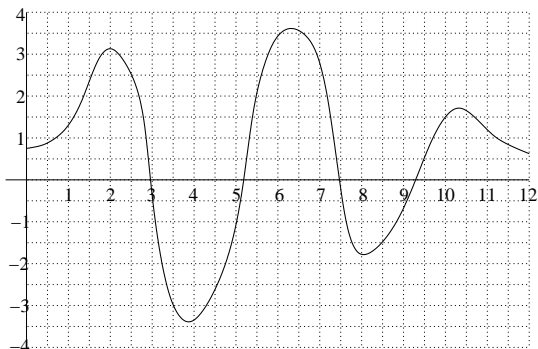
5. Below is the graph of the function $f(x) = x^2 + 2x + 1 + e^x$ together with a tangent line. Find the **exact coordinates** of the point labeled P .



6. Given the graph of $f(x)$ below, sketch a graph of $f'(x)$ as accurately as possible.



7. A particle moves along the x -axis, its position at time t given by $s(t) = e^t - 2t$ meters. Positive values of $s(t)$ indicate that the particle is to the right of $x = 0$, and negative values indicate that the particle is to the left of $x = 0$.
- Calculate and interpret $s(0)$, $s'(0)$, and $s''(0)$ in the context of this problem. Include the proper units with each answer.
 - At what time is the particle moving to the right at 4 meters per second?
8. Given below is $f'(x)$, the graph of the **derivative** of a function $f(x)$.



- On what intervals is $f(x)$ increasing? decreasing?
 - On what intervals is $f'(x)$ increasing? decreasing?
9. If $f(3) = 4$, $g(3) = 2$, $f'(3) = -6$, and $g'(3) = 5$, find the following numbers.
- $(f + g)'(3)$
 - $(f \cdot g)'(3)$
 - $\left(\frac{f}{g}\right)'(3)$
 - $\left(\frac{f}{f - g}\right)'(3)$