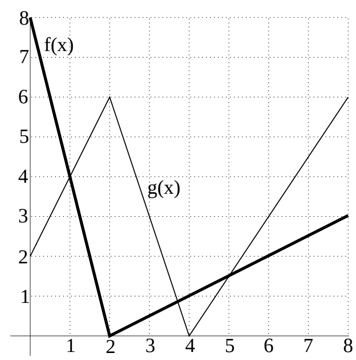
Sample Exam Problems 2

Note. This is not a template. It is likely that topics that don't appear here will appear on the in-class exam. However, these problems are intended to give you an idea of the style and difficulty level of exam questions, as well as to remind you of some of the major topics we have covered.

1. For each of the following, find $\frac{dy}{dx}$. You may assume that a and b are constants. (REMINDER: No calculators would be allowed for an exam question resembling this one.)
   (a) $y = \mathrm{e}^{ax\cos x}$ (b) $y = \sin(\tan(bxe^x))$
   (c) $x\ln(xy) + \mathrm{e}^x = y$ (d) $y = x^{1/x}$

2. If $f(x)$ and $g(x)$ are the functions whose graphs are shown to the right, let $u(x) = f(g(x))$, let $v(x) = g(f(x))$, and let $w(x) = f(f(x))$. Calculate each of the following, if it exists: $u'(6)$, $v'(1)$, and $w'(1)$. If one or more of these derivatives does not exist, explain why.



3. (a) Find the linear approximation for $f(x) = e^x$ at $x = 0$ and use it to approximate $e^{0.2}$.
   (b) Find the linear approximation for $f(x) = \sqrt[3]{1+x}$ at $x = 7$ and use it to approximate $\sqrt[3]{7.9}$.

4. A television camera is positioned 2000 feet away from the launch pad during a space shuttle launch. As the shuttle rises, the camera is kept aimed at the shuttle at all times. When the shuttle is 3000 feet above the ground, it is rising at a rate of 300 feet per second.

   (a) How fast is the distance between the shuttle and the camera increasing when the shuttle is 3000 feet above the ground?

   (b) How fast is the angle between the line of sight of the camera and the ground changing when the shuttle is 3000 feet above the ground?

5. For each of the following, sketch the graph of a function $y = f(x)$ on the interval $0 \le x \le 4$ which has all of the given properties.

   (a) $f(x)$ is continuous everywhere, has a local minimum value of 2 at $x = 1$, a local and absolute maximum value of 6 at $x = 3$, an absolute minimum value at $x = 4$, and $f'(x)$ equals zero at ONLY ONE real number.

   (b) $f(x)$ is continuous everywhere and has a local maximum value that is less than one of its local minimum values.

6. For each of the following functions $f(x)$, use the provided formula for $f'(x)$ and $f''(x)$ to find: (i) the intervals on which $f(x)$ is increasing/decreasing, (ii) the intervals on which $f(x)$ is concave up/concave down, (iii) the local maximum and minimum values of $f(x)$, and (iv) the inflection points of $f(x)$.

   (a) $f(x) = \frac{1}{2}x^2 - 71x + 70\ln x$ Given: $f'(x) = \dfrac{(x-1)(x-70)}{x}$ and $f''(x) = \dfrac{x^2-70}{x^2}$

   (b) $f(x) = \mathrm{e}^{-x}(31 - 4x - x^2)$ Given: $f'(x) = e^{-x}(x^2 + 2x - 35)$ and $f''(x) = e^{-x}(37 - x^2)$

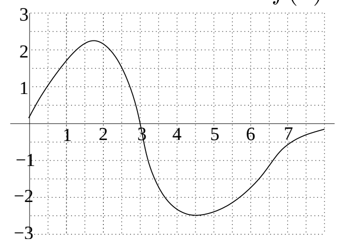
7. Let $f(x) = xe^{-cx} + d$.

   (a) In order for $f(x)$ to have a local maximum value of 2 at $x = 4$, it must be the case that $f'(4) =$ ____________ *and* that $f(4) =$ ____________.

   (b) Find the values of c and d so that $f(x)$ has a local maximum value of 2 at $x = 4$. (**Hint:** Use the two equations from part (a) to solve for c and d.)

8. For each of the following, use the provided graph to estimate the requested information.
   (a) To the right is a graph of $f(x)$.

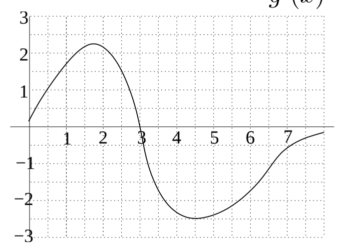
   $f(x)$

      i. Where is $f(x)$ increasing? decreasing?
      ii. Where is $f(x)$ concave up? concave down?
      iii. Locate any local extreme values of $f(x)$.

   (b) To the right is a graph of $g'(x)$, the DERIVATIVE of a function $g(x)$.

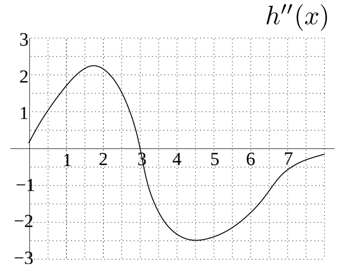
   $g'(x)$

      i. Where is $g(x)$ increasing? decreasing?
      ii. Where is $g(x)$ concave up? concave down?
      iii. Locate any local extreme values of $g(x)$.

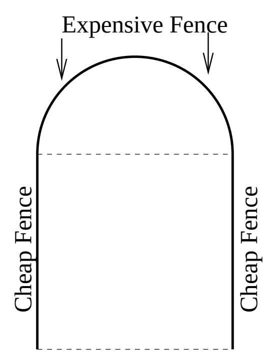
   (c) To the right is a graph of $h''(x)$, the SECOND DERIVATIVE of a function $h(x)$.

   $h''(x)$

      i. Where is $h(x)$ concave up? concave down?

9. A gardener wants to design a two-piece garden in the shape of a rectangle with a semicircular region extending from one side (see diagram to the right). He will plant his prized rutabagas in the semicircular part of the garden, so he will need a special kind of fence costing \$5 per foot along the semicircular border of the garden. He can use a cheaper fence costing \$2 per foot along the other two sides of the yard to keep out those rascally rabbits. (No fence is needed along the dotted lines). Given that the gardener has \$200 total to spend on fencing material, what should the dimensions be so that he gets the largest possible combined area? What is this maximum area?



10. For each of the following, find the critical numbers of $f(x)$. (REMINDER: No calculators would be allowed for an exam question resembling this one.)

    (a) $f(x) = x^2 + \dfrac{8}{x}$
    (b) $f(x) = x\mathrm{e}^{-x^2}$
    (c) $f(x) = (x+1)\ln(x+1)$

11. For each of the following, find the limit.

    (a) $\displaystyle\lim_{x\to\infty} \frac{\ln \ln x}{x}$
    (b) $\displaystyle\lim_{x\to 0^+} \sin x \ln x$