

## Practice Final Exam Problems

**Note.** These problems are intended to give you an idea of the style and difficulty level of exam questions. This is not a template, topics that don't appear here will appear on the final exam, so you should also review problems from old exams and previous practice exam problems to give you an idea of the topics you need to work on, and then do more practice problems relating to those topics.

1. Find  $\frac{dy}{dx}$  in parts (a) - (h) below.

(a)  $y = e^2 + e^{2x}$

(c)  $y = x \arctan x$

(e)  $y = \cos \sqrt{1+x^2}$

(g)  $y = \int_2^x \sqrt{1+t^3} dt$

(i)  $y = \int_{-2}^{\cos x} \sqrt{1-t^2} dt$

(b)  $y = x^e \cdot e^x$

(d)  $y + \ln(xy) = 1$

(f)  $y = \frac{2 + \ln x}{2 - \ln x}$

(h)  $y = xg(x^2)$

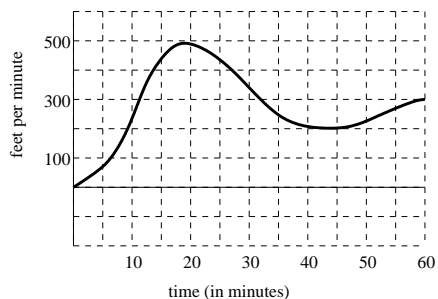
(j)  $x^{x^2}$

2. Find the equation of the tangent line to  $y = x \ln x$  at  $x = e$ .
3. Use the **limit definition** of the derivative to find a formula for  $f'(x)$  given that

(a)  $f(x) = \frac{1}{2+x}$

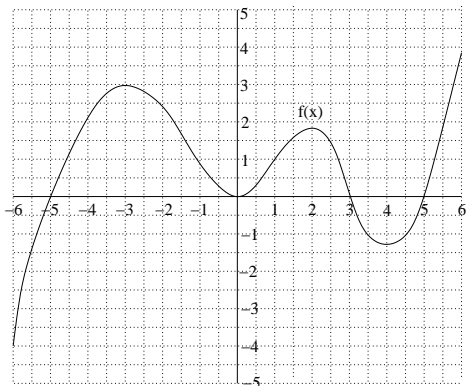
(b)  $f(x) = \sqrt{x}$

4. Find the **exact** area enclosed by the graph of  $y = -e^x + 5$ , the  $x$ -axis, and the  $y$ -axis.
5. A helicopter flies along a north-south straight line. Throughout the flight, its velocity is recorded. Velocity to the north is recorded as positive and to the south is recorded as negative. The velocity is graphed below.

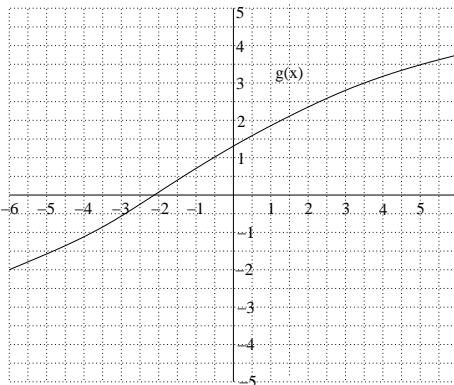


- (a) Does the helicopter reverse its direction during the recorded flight? If so, at what approximate time(s)?
- (b) Does the helicopter change its velocity during the recorded flight? If so, approximately when is it flying slowest and when is it flying fastest?
- (c) Does the helicopter change its acceleration during the recorded flight? If so, approximately when is it accelerating the most?
- (d) Does the helicopter ever decelerate during the recorded flight? If so, approximately when is it decelerating the most?
- (e) Approximately how far does the helicopter fly during the first 20 minutes of its flight?
6. At 2 p.m., a 747 passenger jet flying east at 600 miles per hour passes directly over a biplane flying west at 100 miles per hour. The altitude of the jet and the biplane are 6 miles and 2 miles, respectively. Assuming that both planes are flying level at constant speeds, how fast is the (diagonal) distance between the planes increasing at 2:30 p.m.?

7. Let  $f(x)$  be the function whose graph is given below, and let  $F(x) = \int_0^x f(t) dt$ .



- (a) What are the critical points of  $F(x)$ ?
- (b) Where is  $F(x)$  increasing? Where is  $F(x)$  decreasing?
- (c) For what values of  $x$  does  $F(x)$  have a local minimum? a local maximum?
- (d) Where is  $F(x)$  concave down?
- (e) Estimate the value of each of the following quantities:  $F(3)$ ,  $F'(3)$ ,  $F''(3)$ ,  $\int_2^4 f'(x) dx$
8. Below, you are given two functions. Use the midpoint rule with  $n = 4$  to estimate the two integrals that follow.



$$f(x) = 1 - \ln x$$

$$\int_1^5 f(x) dx$$

$$\int_{-6}^6 g(x) dx$$

9. Evaluate each of the following definite and indefinite integrals, giving exact answers in each case. No credit will be given if work is not shown.

(a)  $\int x^2(x^3 + 1)^{10} dx$

(b)  $\int \sin \theta \sqrt{1 + 3 \cos \theta} d\theta$

(c)  $\int_e^{e^2} \frac{1}{x(\ln x)^2} dx$

(d)  $\int_0^1 xe^{3x^2+1} dx$

10. Suppose the line  $y = 5x - 4$  is tangent to the curve  $y = f(x)$  when  $x = 3$ . If Newton's method is used to locate a root of the equation  $f(x) = 0$  and the initial approximation is  $x_0 = 3$ , find the second approximation  $x_1$ .

11. Sketch the graph of one function  $f(x)$  defined on the interval  $[-4, 4]$  which simultaneously has all of the following properties:

- (a)  $f$  is continuous and differentiable on the entire interval  $[-4, 4]$ .
- (b)  $f'(x) < 0$  for  $-4 < x < -3$  and for  $3 < x < 4$
- (c)  $f'(x) > 0$  for  $-3 < x < 0$
- (d)  $f''(x) > 0$  for  $-4 < x < -2$  and for  $0 < x < 2$
- (e)  $f''(x) < 0$  for  $-2 < x < 0$  and for  $2 < x < 4$

12. A rectangular swimming pool is to be built with an area of 2000 square feet. The owner wants 2-foot wide decks along either side and 4-foot wide decks at the two ends (see diagram to the right). Find the dimensions of the smallest piece of property (that is, the smallest combined area of the pool and the decks) on which the pool can be built satisfying these conditions.

